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CS-C2160 Theory of Computation — Spring 2022

Exam Mon 11 April 2022, 5–8 p.m.

You are allowed to bring with you a single one-sided A4 "cheat sheet", **personally handwritten by you**. (**NO photocopies, NO printouts, NO computer type-set text.**) Please include your name and student ID at the top of the cheat sheet, and return it together with your answer sheets at the end of the exam.
Note: If you have not completed your computerised home assignments, your exam will not be graded.

1. Design **deterministic** finite automata for recognising the following languages:

   (a) $\{w \in \{0,1\}^* \mid w$ does not contain the substring 1011$\}$,

   (b) $\{w \in \{0,1\}^* \mid w$ begins and ends with different symbols$\}$,

   (c) $\{w \in \{0,1\}^* \mid$ the difference in the number of 0's and 1's in w is a multiple of 3$\}$ (here also zero is considered to be a multiple of 3),

   (d) $\{w \in \{0,1\}^* \mid w$ contains an even number of 1's and the substring 110$\}$.

   16 points

2. (a) Consider the following context-free grammar G:

   $$S \to aSb \mid aSbb \mid \varepsilon$$

   Describe the language $L = L(G)$ generated by G mathematically or verbally as simply as you can. Show that G is ambiguous.

   (b) Design an unambiguous grammar equivalent to G, that is an unambiguous G' for which $L(G') = L$. (You do not need to prove the unambiguity of G'.)

   (c) Prove (precisely!) that the language $L = L(G)$ is not regular. 15 points

3. *One* of the following:

   (a) Show that if a language L is context-free then so are the following languages:

   $$\begin{aligned} L^* &= \bigcup_{k \geq 0} L^k = \{w_1 \ldots w_k \mid k \geq 0,\ w_i \in L \text{ for all } i = 1, \ldots, k\}, \\ L^R &= \{w^R \mid w \in L\}. \end{aligned}$$

   (Here w^R denotes the *reversal* of string w, i.e. w written backwards.

   (b) Show that if languages L_1 and L_2 are semidecidable, then so are the languages $L_1 \cap L_2$ and $L_1 L_2 = \{w_1 w_2 \mid w_1 \in L_1, w_2 \in L_2\}$.

   15 points

PLEASE TURN OVER!

4. Which of the following claims are true and which are false? (No proofs are needed, just indicate your choice by the letter T or F.)

   (a) The language $\{ww^R \mid w \in \{0,1\}^*\}$, where w^R denotes the reversal of string w, can be recognised by a nondeterministic finite automaton.

   (b) The problem of determining if two nondeterministic finite automata are equivalent, i.e. recognise the same language, is decidable.

   (c) The problem of determining if a given string is *not* generated by a context-free grammar is decidable.

   (d) Every language that can be recognised by a deterministic pushdown automaton can be described by a regular expression.

   (e) The problem of determining if a Turing machines halts on some input is semidecidable.

   (f) The complement of any semidecidable language is also semidecidable.

   (g) Some undecidable languages can be recognised by nondeterministic Turing machines.

14 points

Total 60 points