1. (Multiple-choice problems on the separate paper.)

2. Answer concisely (but informatively) the following in words.
   In case you need, you can use also equations to support your answer.
   (a) Assume a scalar field $V(\mathbf{R})$ that satisfies Laplace's equation $\nabla^2 V = 0$. Describe the properties of this function.
   (b) For solving dynamic fields due to time-varying sources, we used so-called *retarded potentials.* What are retarded potentials?
   (c) How have we defined the handedness of the polarization of a plane wave in this course (the electrical engineering definition)?

3. For static fields (as time derivatives vanish), the electric fields created by charges are curl-free (in other words: $\nabla \times \mathbf{E} = 0$).
   As you know, the field by a **monopole** (point charge q) located in the origin has the electric field function
   $$\mathbf{E}_\mathrm{m}(\mathbf{R}) = \frac{q}{4\pi\varepsilon_0 R^2}\,\mathbf{a}_R$$
   and the field by an electric **dipole** with moment $\mathbf{p} = p\,\mathbf{a}_z$ at the origin reads
   $$\mathbf{E}_\mathrm{d}(\mathbf{R}) = \frac{p}{4\pi\varepsilon_0 R^3}\,(2\cos\theta\,\mathbf{a}_R + \sin\theta\,\mathbf{a}_\theta)$$
   Show (by explicitely computing the curls) that $\nabla \times \mathbf{E}_\mathrm{m} = 0$ and $\nabla \times \mathbf{E}_\mathrm{d} = 0$, for $R \neq 0$.

4. An electromagnetic plane wave with frequency 2 GHz propagates into the earth ground. For this dry sandy material, the real part of the permittivity is $\varepsilon' = 3\varepsilon_0$, and the conductivity $\sigma = 0.01$ S/m (sand is practically non-magnetic ($\mu = \mu_0$)).
   Since the medium is lossy, the wave will be attenuated. In other words, the dependence of the electric field in space which is
   $$\mathrm{e}^{-\mathrm{j}k_c z}$$
   has a *complex* propagation factor $k_c = \omega\sqrt{\mu_0\varepsilon}$.
   (a) Compute the complex relative permittivity $\varepsilon_r = \varepsilon_r' - \mathrm{j}\varepsilon_r''$.
   (b) How many decibels is the attenuation per meter?

   (Soil at these frequencies should be a fairly low-loss material. In case you do not want to do the exact calculation of the square root of complex numbers, you can use the expansion $\sqrt{1+x} \approx 1 + x/2$ which is valid when $|x| \ll 1$, even if x is a complex number.)

This first problem of the exam has six multiple-choice questions.
Choose, for each question, **one** and **only one** of the answers.
No need to justify your answer.

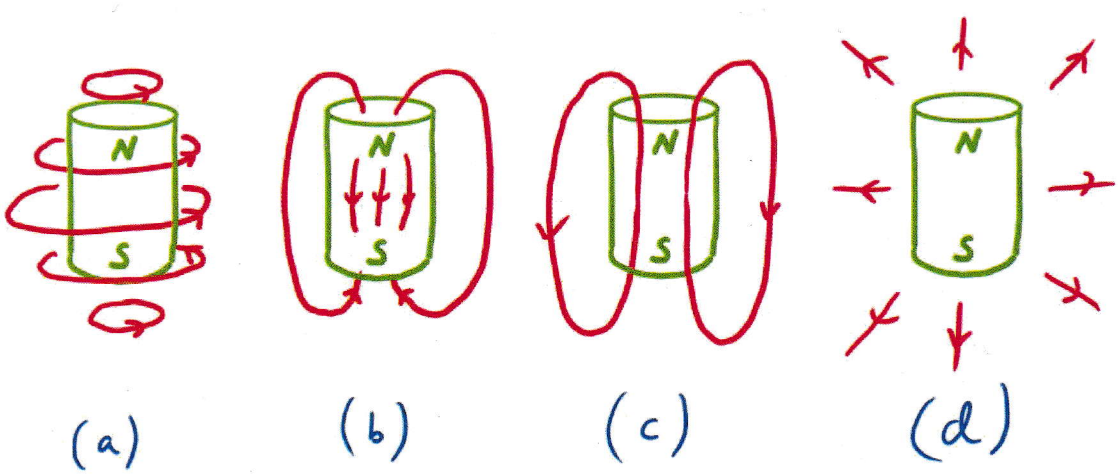
1. Volt (V) is the unit for the electric potential. What is V expressed using the SI units (kg, m, s, A)?

   (a) $\frac{\mathrm{kgm^2}}{\mathrm{As^3}}$ (b) $\frac{\mathrm{A^2s}}{\mathrm{m^2}}$ (c) $\frac{\mathrm{kgm}}{\mathrm{As^2}}$ (d) $\frac{\mathrm{kg^2}}{\mathrm{ms^2}}$ (e) $\frac{\mathrm{Akg}}{\mathrm{ms^2}}$ (f) $\frac{\mathrm{Am^2}}{\mathrm{kgs}}$ (g) something else

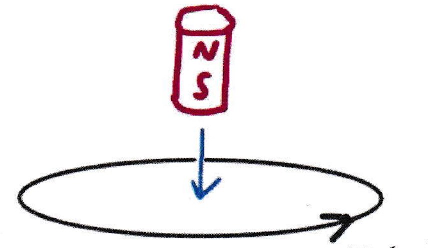
2. The unit vector $\mathbf{a}_\phi$ of the spherical coordinate system can be written in terms of unit vectors in other systems. What is $\mathbf{a}_\phi$ on point $(x, y, z) = (1, -1, 0)$ m of the Cartesian coordinate system?

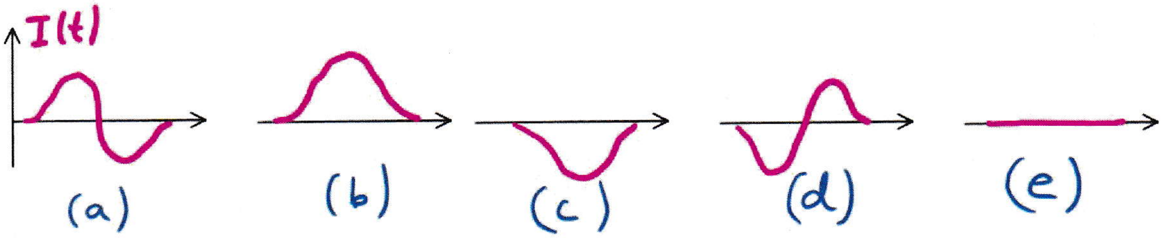
   (a) $-\mathbf{a}_z$
   (b) $\mathbf{a}_x + \mathbf{a}_y$
   (c) $\mathbf{a}_x - \mathbf{a}_y$
   (d) $(\mathbf{a}_x + \mathbf{a}_y)/\sqrt{2}$
   (e) $(\mathbf{a}_x - \mathbf{a}_y)/\sqrt{2}$
   (f) something else

3. Which of the following field line distributions describes best the magnetic field lines of a bar magnet?



4. A bar magnet drops through a conducting circular wire loop. What is the behavior of the current that is induced in the loop?
   (Note the definition of the sign of the current $I(t)$ as in the figure: it is positive when it is counterclockwise when looked from above the loop.)





5. The electric field of a plane wave propagating in free space has the following real, time-dependent form:

$$\mathbf{E}(x, t) = \mathbf{a}_z E_0 \cos(\omega t - kx)$$

Its magnetic field $\mathbf{H}(x, t)$ reads (with $H_0 = E_0/\eta_0$)

(a) $+\mathbf{a}_y H_0 \cos(\omega t - kx)$

(b) $-\mathbf{a}_y H_0 \cos(\omega t - kx)$

(c) $+\mathbf{a}_y H_0 \cos(\omega t + kx)$

(d) $-\mathbf{a}_y H_0 \cos(\omega t + kx)$

(e) $+\mathbf{a}_y H_0 \sin(\omega t - kx)$

(f) $-\mathbf{a}_y H_0 \sin(\omega t - kx)$

(g) $+\mathbf{a}_y H_0 \sin(\omega t + kx)$

(h) $-\mathbf{a}_y H_0 \sin(\omega t + kx)$

(i) something else

6. A z-directed Hertzian dipole is located in the origin. In its far-field at the z-axis, the electric field is of the form:

(a) $\mathbf{E} = E_x \mathbf{a}_x$

(b) $\mathbf{E} = E_y \mathbf{a}_y$

(c) $\mathbf{E} = E_z \mathbf{a}_z$

(d) $\mathbf{E} = E_x \mathbf{a}_x + E_y \mathbf{a}_y$

(e) $\mathbf{E} = E_x \mathbf{a}_x + E_y \mathbf{a}_y + E_z \mathbf{a}_z$

(f) $\mathbf{E} = \mathbf{0}$