Aalto University

MS-A0402 Foundations of Discrete Mathematics

Final exam, 14.04.2022 at 9.00–12.00

Write your name and student number on each page.

There are in total 5 questions. **Calculators, phones, computers or any notes, sheets, etc. are strictly forbidden**. *Answers must always be justified with calculations, explanations and clear logic so the person marking can follow your argument. For example, it can be helpful to use implication arrows ⇒, words "Thus", "Therefore", "Hence" etc., or short sentences that make it clear what the logic in the answer is, especially in the proofs.*

On the grading:

1. *If you have not done any assignments, then 100% of your final grade comes from the marks you get from this exam (from all 5 questions)*
2. *If you have assignment/exercise marks (like most students taking this exam), there are two ways your final grade is determined:*
   (a) *Firstly, we will look at 4 of your answers that earn the most points out of all 5 and this will determine 40% of your grade. The rest 60% will come from the assignment/exercise marks done during the course.*
   (b) *Secondly, we will grade you as we would in case 1, that is,* ***we will also look at what your mark would be if all 5 (including the answer with the lowest mark) are graded*** *and what mark you would get if we do not take assignment/exercise marks into account.*

   Then the ***better*** *(max) of these two performances in (a) and (b) will determine your final grade. Therefore, I recommend to* ***attempt all 5****, even writing down some ideas on them if you are unsure how to start.*

Question 1. a) (2pts) Define the *cartesian product* $S \times T$ of sets S and T.

b) (2pts) Define the *Power set* $P(A)$ of a set A.

c) (2pts) Let $S = \{a, b\}$ and $T = \{1, 2\}$. Write down

$$P(S \times T).$$

c) (4pts) Prove that for all non-empty sets A, B, C we have

$$A \times (B \cup C) \subset (A \times B) \cup (A \times C).$$

Question 2. a) (5pts) Prove that for all odd integers n, then n^2 is also odd.

b) (5pts) Prove, using induction, that for all $n = 1, 2, 3, \dots$,

$$\sum_{i=1}^{n} (2i - 1) = n^2.$$

Question 3. a) (3pts) Write the statement of the *Binomial theorem.*

b) (3pts) Write down the first 6 rows of the Pascal's triangle.

c) (4pts) What is the coefficient of x^2y^3 in the expansion of $(x + y)^5$?

Question 4. a) (3pts) Write the permutation $(1362)(2564)(2345)$

i) as a product of disjoint cycles.

ii) in two-line notation.

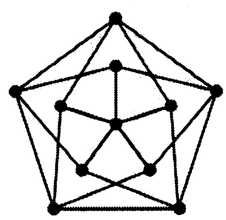
iii) as a product of transpositions.

b) (7pts)

i) There are 6 participants who are first paired up for a dance. Afterwards, they pair up to play a game, where for some reason it is important that the two people in each pair did not dance with each other. In how many ways can this be done?

ii) Solve the same problem when there were initially $2n$ participants for $n = 1, 2, 3, \dots$

Question 5. a) (5pts) Compute (with proof) the chromatic number of the Grötszch graph, depicted below.



b) (5pts) Find all the solutions $x, y \in \mathbb{Z}$ for the Diophantine equation

$$514x + 387y = 2.$$