1. Multiple-choice problems on the separate paper.

2. Answer concisely (shortly but informatively) the following in words.
   (In case you need, you can use equations/drawings in addition.)

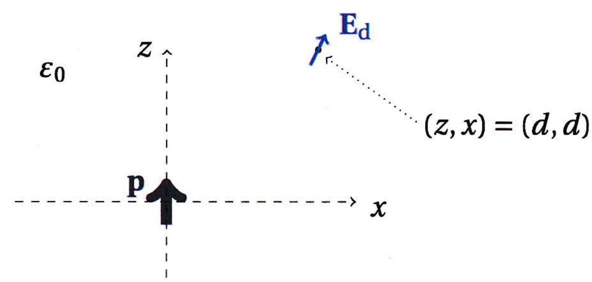
   (a) The field of a charge above a conducting plane can be solved using the method of images (image principle): replacing the conducting plane by a negative charge at the image point. Why does this principle work?

   (b) Assume a scalar field $V(\mathbf{R})$ that satisfies Laplace's equation $\nabla^2 V = 0$. Describe the properties of this function.

   (c) Explain *Faraday's law.*

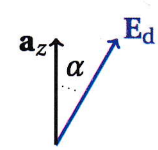
3. An **electrostatic** dipole with dipole moment p is located in the origin of free space and it is directed along the z-axis: $\mathbf{p} = \mathbf{a}_z\, p$. As you know, the scalar potential due to this dipole is

$$\phi_{\mathrm{d}}(R,\theta,\phi) = \frac{p\cos\theta}{4\pi\varepsilon_0 R^2}$$



   (a) Derive the expression for the electric field $\mathbf{E}_{\mathrm{d}}$ due to this dipole. (Remember that in electrostatics, the field is the negative gradient of the potential.)

   (b) Consider the electric field vector at the point $(z,x) = (d,d)$. Calculate the angle α that the electric field vector $\mathbf{E}_{\mathrm{d}}$ makes with the z-axis.



4. A plane wave propagates in free space into positive z-direction with the field function

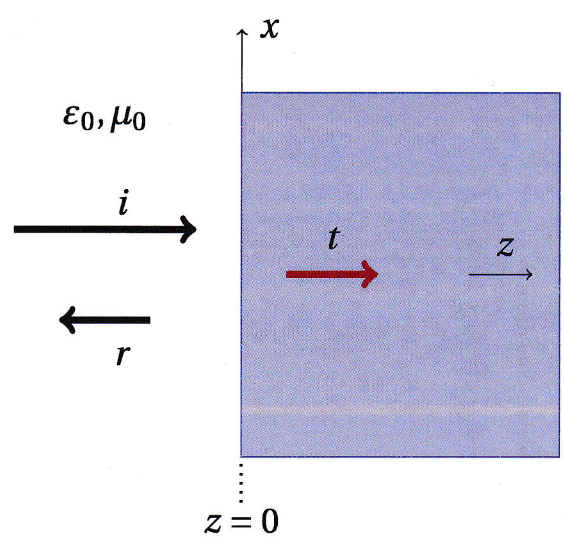
$$\mathbf{E}_i(z) = \mathbf{a}_x E_0 \mathrm{e}^{-\mathrm{j}kz}$$

   It hits a planar boundary at the xy-plane ($z = 0$), and there will be reflection. In other words, a reflected field $\mathbf{E}_r$ is generated. The reflection coefficient is $\Gamma = 1/2$.
   (Here $k = \omega\sqrt{\mu_0\varepsilon_0}$ and you can assume E_0 to be real.)

   (a) Write the expression of the total electric field $\mathbf{E}_{\text{total}} = \mathbf{E}_i + \mathbf{E}_r$ in the air region ($z < 0$).

   (b) Compute the complex Poynting vector $\frac{1}{2}\mathbf{E}_{\text{total}} \times \mathbf{H}^*_{\text{total}}$ in the same region $z < 0$.

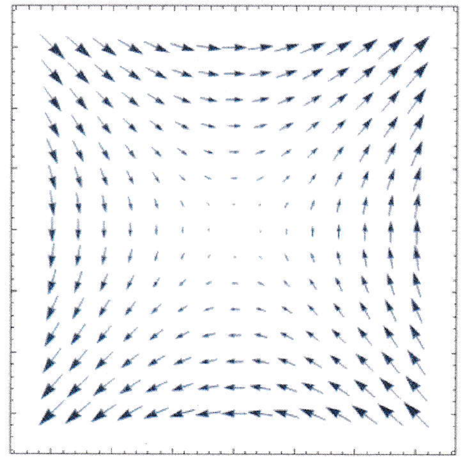
   (c) Analyze and interpret your result for the complex Poynting vector. What can you say about the character of the total field?



This first problem of the exam has six multiple-choice questions.
Choose, for each question, **one** and **only one** of the answers.
No need to justify your answer.

1. Consider the vector function

$$\mathbf{F} = y\mathbf{a}_x + x\mathbf{a}_y$$



What can you say about its divergence $\nabla \cdot \mathbf{F}$ and curl $\nabla \times \mathbf{F}$?

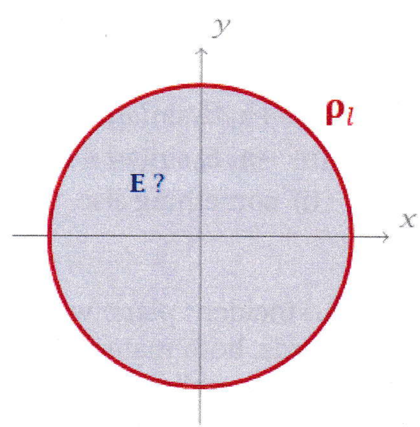
(a) $\nabla \cdot \mathbf{F} = 0$ and $\nabla \times \mathbf{F} = 0$

(b) $\nabla \cdot \mathbf{F} = 0$ and $\nabla \times \mathbf{F} \neq 0$

(c) $\nabla \cdot \mathbf{F} \neq 0$ and $\nabla \times \mathbf{F} = 0$

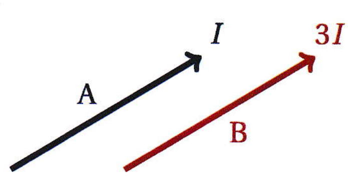
(d) $\nabla \cdot \mathbf{F} \neq 0$ and $\nabla \times \mathbf{F} \neq 0$

2. A circular line charge ($\rho_l > 0$) generates a static electric field. What can you say about the direction of the electric field in the plane of the charge (the blue region), inside the circular charge? Using unit vectors in the cylindrical coordinate system, the field is pointing into



(a) $+\mathbf{a}_r$
(b) $+\mathbf{a}_\phi$
(c) $+\mathbf{a}_z$
(d) $-\mathbf{a}_r$
(e) $-\mathbf{a}_\phi$
(f) $-\mathbf{a}_z$
(g) some other direction
(h) cannot be concluded without additional information about the problem

3. Two parallel wires A and B that are near to each other carry currents I and $3I$ in the same direction. Compare the forces that the two current wires exert on each other.



(a) Wire A exerts a stronger force on wire B than B exerts on A.
(b) Wire B exerts a stronger force on wire A than A exerts on B.
(c) The wires exert equal-magnitude attractive forces on each other.
(d) The wires exert equal-magnitude repulsive forces on each other.
(e) The wires exert no forces on each other.

4. Plane wave A has peak electric field amplitude 2 V/m.
Plane wave B has peak electric field amplitude 40 V/m.

How much stronger is wave B compared to wave A?

(a) 1 dB
(b) 3 dB
(c) 6 dB
(d) 10 dB
(e) 16 dB
(f) 26 dB
(g) 42 dB
(h) 60 dB
(i) 68 dB

5. The magnetic field of a plane wave propagating in free space has the following real, time-dependent form:

$$\mathbf{H}(x,t) = \mathbf{a}_z H_0 \cos(\omega t + kx)$$

Its electric field $\mathbf{E}(x,t)$ reads (with $E_0 = \eta_0 H_0$)

(a) $+\mathbf{a}_y E_0 \cos(\omega t - kx)$
(b) $-\mathbf{a}_y E_0 \cos(\omega t - kx)$
(c) $+\mathbf{a}_y E_0 \cos(\omega t + kx)$
(d) $-\mathbf{a}_y E_0 \cos(\omega t + kx)$
(e) $+\mathbf{a}_y E_0 \sin(\omega t - kx)$
(f) $-\mathbf{a}_y E_0 \sin(\omega t - kx)$
(g) $+\mathbf{a}_y E_0 \sin(\omega t + kx)$
(h) $-\mathbf{a}_y E_0 \sin(\omega t + kx)$
(i) something else

6. An incident plane wave encounters a planar interface between two dielectric materials (in other words, both materials have equal permeability μ_0 but their permittivities are different). The incidence is oblique: the incidence angle $\theta_i \neq 0$. Which one of the two polarizations is reflected more strongly?

(a) parallel polarization
(b) perpendicular polarization
(c) both reflections are equally strong
(d) cannot be concluded without additional information about the problem