PHYS-C0256 - Thermodynamics and Statistical Physics
Exam on April 14, 2022

6 problems - 30 points

1. Suppose we have a box with two chambers. In each of the two chambers we have the same ideal gas. Which conditions need to be fulfilled by (1.5 point)
(i) thermal equilibrium,
(ii) chemical equilibrium,
(iii) mechanical equilibrium?

2. (i) A body of constant heat capacity C, and a temperature T_i is put into contact with a reservoir at temperature T_f. Equilibrium between the body and the reservoir is established at constant pressure. Determine the total entropy change of the whole system. (3 points)

(ii) Based on the definition for C_v and C_p and using Maxwell's relations show that (3 points)

$$\left(\frac{\partial C_v}{\partial V}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V. \tag{1}$$

3. A Carnot engine has a cycle as shown in Fig. 1. If W and W' represent work done by one mole of monatomic and diatomic gas, respectively, calculate W'/W. (For a monatomic gas $\gamma = 5/3$; for a diatomic gas, $\gamma' = 7/5$.) (3.5 points)
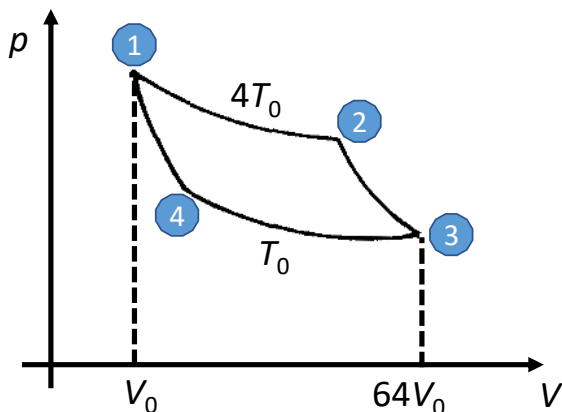

FIG. 1. Setup for problem 3.

4. Consider a spin-1/2 system in form of a paramagnet with energy levels $E = -m\bar{\mu}B$, where m assumes values $-1/2$ and $+1/2$. B is the magnetic field and $\bar{\mu}$ the effective magnetic moment. Find for a system of N independent such systems:
(i) the internal energy U (2 points)
(ii) the heat capacity C_B at a constant field B. (2 points)
(iii) Write down the time dependent temperature of this system when it is heated with power P assuming it is at temperature T_0 initially. (3 points)

5. Consider a normal metal-insulator-normal metal NIN junction;
(i) Find an analytical expression for the voltage V dependence of tunneling rates in a junction between two leads at the same temperature T. Show that the detailed balance condition for the forward and reverse rates holds, i.e. (3 points)

$$\Gamma_{\text{reverse}} = e^{-eV/k_B T}\Gamma_{\text{forward}}. \tag{2}$$

(ii) Show that the power into both electrodes is the same and half of the total power V^2/R_T, where R_T is the resistance of the junction. (3 points)

6. A quantum two-level system (qubit) with energy level separation E is coupled to a heat bath. This coupling gives the transition rates $\Gamma_\uparrow$ from ground state $|g\rangle$ to the excited state $|e\rangle$, and $\Gamma_\downarrow$ from the excited state to the ground state.

The qubit is prepared in the superposition state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$ by an external pulse at the initial time instant $t = 0$. Find:

(i) the time dependent population of the qubit in the excited state ρ_{ee} (3 points)

(ii) and the off-diagonal element ρ_{ge} of the density matrix. (3 points)