ELEC-C9440 Quantum Information Exam 2.6.2022

Problem 1

Consider the two qubit state described by

$$\rho = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \in \mathcal{H}_A \otimes \mathcal{H}_B \, .$$

a)

Is the state pure?

b)

Are the two qubits entangled?

Problem 2

Show that the n-qubit quantum Fourier transform

$$U_{QFT} = \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} \sum_{k=0}^{2^n-1} e^{-2\pi i jk/2^n} \, |k\rangle\langle j|$$

is unitary. That is, show that

$$U_{QFT}^\dagger U_{QFT} = \mathbb{I}_{2^n} = \sum_{k=0}^{2^n-1} |k\rangle\langle k| \, .$$

Problem 3

Describe the error correction protocol for the Shor 9-qubit code including (i) encoding, (ii) syndrome measurements and (iii) recovery operations.

Problem 4

Provide a quantum circuit for simulating one timestep of the dynamics of a chain of three interacting spin-$\frac{1}{2}$ particles (modelled by single qubits) with the Hamiltonian

$$H = \mu \sum_{i=1}^{3} \sigma_x^{(i)} + \lambda \sigma_x^{(1)} \sigma_y^{(2)} \sigma_z^{(3)} \, ,$$

where $\sigma_k^{(i)}$ is the k'th Pauli matrix acting on the i'th particle.

Problem 5

Suppose you want to measure the expectation value of some observable M in a quantum state you prepare on a quantum computer. That is, you are interested in $\langle M \rangle$ computed in some state $|\psi\rangle$.

a)

What is quantum error mitigation and why would one want to use it instead of quantum error correction for this computation?

b)

Assume your quantum circuit which prepares $|\psi\rangle$ consists of only single qubit gates and CNOT gates. Further assume that the error rate of single qubit gates is zero and the all CNOT gates have an error rate ϵ.

You have measured $\langle M \rangle$ at two different error rates:

$$\langle M \rangle (\epsilon) = x$$
$$\langle M \rangle (3\epsilon) = y \ .$$

Calculate the estimate for $\langle M \rangle (0)$ using Richardson extrapolation.

c)

Give one method for increasing the error rate ϵ in the measurement of $\langle M \rangle$.