Differential and Integral Calculus 1
MS-A0111
Hakula / Orlich
Exam, Aug 31, 2022

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Please follow the instruction given on the exam page. Every question carries an equal weight, similarly every part of a question carries an equal weight, unless otherwise specified. You are not allowed to use a calculator, tables or notes.

PROBLEM 1 Find the derivative of $f(x) = \tanh x$ using Newton's Quotient.

PROBLEM 2 a) Find the limits

$$\lim_{x\to 0} \frac{e^{2x}-1}{x} \quad \text{and} \quad \lim_{x\to 1} \frac{(\ln x)^2}{(x-1)^2}.$$

b) Find the Maclaurin polynomial of degree two for the function $f(x) = e^{2x}$.

PROBLEM 3 Consider Newton's method for solving $f(x) = 0$.
a) Explain how the recurrence formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

is derived graphically with the help of the tangent line. (4p)
b) Compute one approximation x_1 for $x^4 - 2 = 0$ given $x_0 = 2$. (2p)

PROBLEM 4 Compute the integrals

$$\int_0^2 20(x-2)^4\,dx \quad \text{and} \quad \int_0^{\pi^2} \sin(\sqrt{x}\,)\,dx.$$

Hint: Sometimes $x = u^2$ is a powerful substitution.

PROBLEM 5 Solve $y' = 6xy^3$ given the initial values
a) $y(0) = 1$;
b) $y(0) = 0$.

PROBLEM 6 Find the complete solution.

$$\begin{cases} y'' - 3y' + 2y = 10\sin x, \\ y(0) = 0, \\ y'(0) = 0. \end{cases}$$

Formulae:

α	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin(\alpha)$	$-1/\sqrt{2}$	$-1/2$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0
$\cos(\alpha)$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1
$\tan(\alpha)$	-1	$-1/\sqrt{3}$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$-$	0

More:

$$D\arcsin x = \frac{1}{\sqrt{1-x^2}}, \qquad D\arctan x = \frac{1}{1+x^2}$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}, \qquad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$$

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k, \qquad \ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \ldots$$