ELEC-E8101 Digital and Optimal Control

Final Exam (04.05.2020)

- *All four (4) problems must be answered.*
- *You can write the solution as you wish in* ***one pdf*** *file, which contains the solutions such that it can be seen how you have solved the problems. You can use handwriting, but the result must be well readable. Please keep your solutions just like you made and submitted them, in case something went wrong with the submission or need to be checked afterwards.*
- *The exam is "open book" so you can have the course material (lectures, exercises, homework, Databook, etc) at your disposal. But you must not take help from any other person while taking the exam.*
- *Electronic calculators and computer software can be used, but* ***plots (if any) should be drawn in hand and justified****.*
- *With your signature (on the answer sheet you will submit) you assert that you have followed the above regulations.*

Good luck!

(Turn over)

1. Consider the open loop discrete time system shown in Figure 1

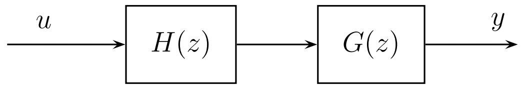

Figure 1: Open loop discrete time system.

with transfer functions

$$H(z) = \frac{1-3z}{3(1-z)} \quad \text{and} \quad G(z) = \frac{6}{1-6z}.$$

a) Is the open loop system Bounded-Input-Bounded-Output (BIBO) stable? Justify your answer. [1p]

b) Compute the pulse response of the open loop system. [2p]
Reminder: The equation of a pulse is given by:

$$u[k] = \begin{cases} 1, & \text{for } k = 0, \\ 0, & \text{otherwise.} \end{cases}$$

c) Verify the final value theorem for the pulse response of the open loop system. [1p]

d) Explain why the existence of a final value for a pulse input is consistent with your answer to part (a). [1p]

e) Now, consider the closed loop discrete time system shown in Figure 2.

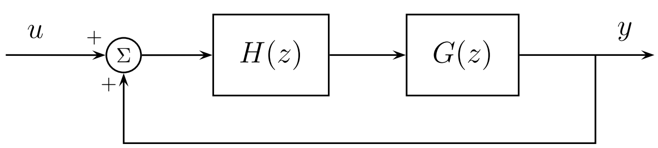

Figure 2: Closed loop discrete time system.

i) Compute the transfer function of the closed-loop system. [2p]

ii) Is the closed-loop system stable? [1p]

iii) Compute the pulse response of the closed-loop system. [2p]

2. Modern subwoofers[1] typically operate in the frequency range 20 − 200 Hz and use filtering and feedback to improve acoustic performance. The dynamics of a subwoofer are captured by the continuous transfer function

$$\bar{y}(s) = G_c(s)\bar{v}(s)$$

from input voltage $v(t)$ to cone position $y(t)$, given by

$$G_c(s) = \frac{10^5}{s(s^2 + 50s + 10^5)}.$$

a) For the sampling period $h = 10^{-3}$, using *backward difference* on $G_c(s)$ derive the corresponding digital transfer function $G_d(z)$. [2p]

b) Is the sampling period $h = 10^{-3}$ sufficient to avoid aliasing for real signals band-limited between 20 − 200 Hz? What is the maximum admissible sampling period? [2p]

c) The subwoofer has a resonance around 50Hz, as shown in Figure 3

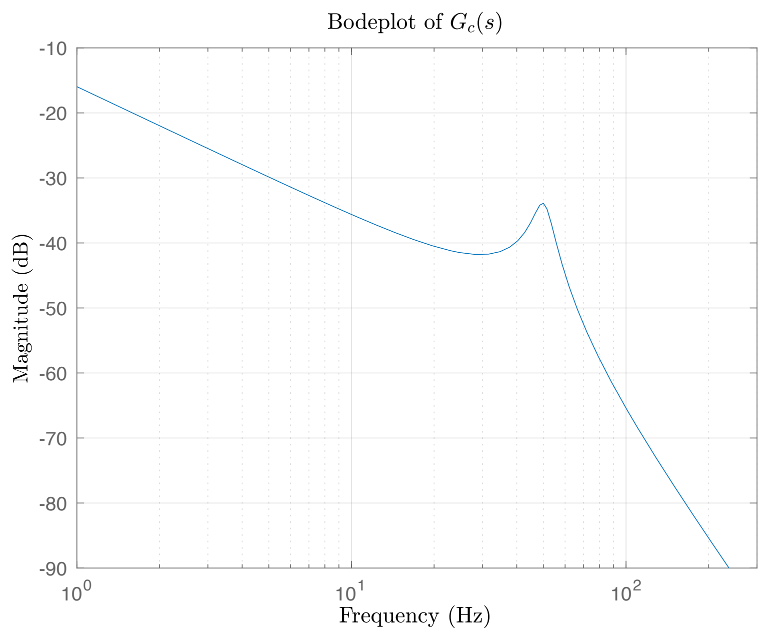

Figure 3: Frequency response of $G_c(s)$.

Show that the resonance can be removed, improving acoustic performance, by prefiltering every signal to the subwoofer with the filter. [3p]

$$H(z) = 10\frac{(z - 0.98e^{j0.1\pi})(z - 0.98e^{-j0.1\pi})}{z^2}$$

[1] A subwoofer is a loudspeaker designed to reproduce low-pitched audio frequencies known as bass and sub-bass, lower in frequency than those which can be (optimally) generated by a woofer.

d) Servo-subwoofers use feedback to improve performance. Consider the proportional feedback controller

$$v[k] = k_p(r[k] - y[k])$$

where r is the reference input and k_p is the proportional gain.

(i) Sketch the compleye Nyquist diagram of $G_d(z)$ in Figure 4. [1p]

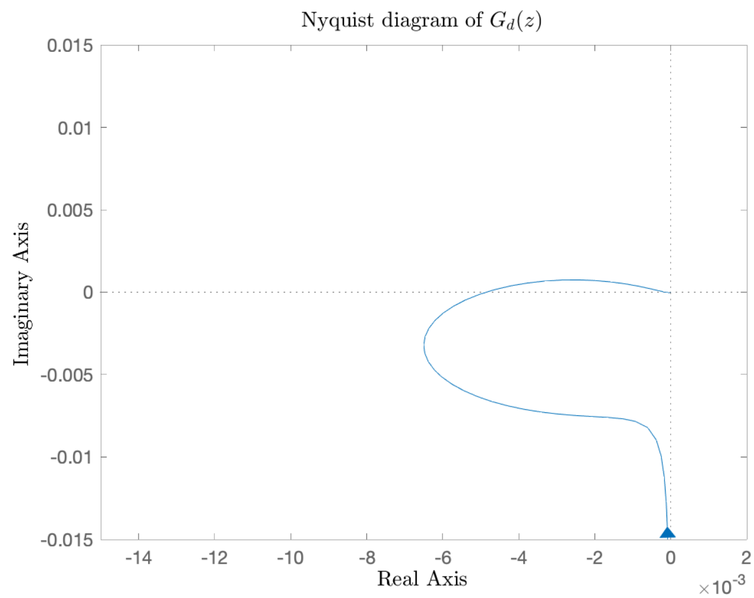

Figure 4: Nyquist diagram of $G_d(z)$.

(ii) Use the Nyquist diagram to determine the range of gains k_p that guarantee closed loop stability. [2p]

3. Consider the discrete-time signal $y[k]$ with transfer function

$$Y(z) = \frac{z}{(z-1)(z-2)}$$

a) Find the signal $y[k]$. [2p]

b) Could the final value theorem be used to find the value of $y[k]$ as $k \to \infty$? Justify your answer. [1p]

c) The signal $y[k]$ above is the output of a linear time invariant system when its input $u[k]$ is the unit step, i.e.,

$$u[k] = \begin{cases} 0, & \text{if } k < 0, \\ 1, & \text{if } k \geq 0. \end{cases}$$

i) Find the transfer function $G(z)$ of this system. [1p]

ii) Find a difference equation that can represent it. [2p]

d) Consider the feedback interconnection below, in which $G(z)$ is the transfer function found in c) i) and K is a constant.

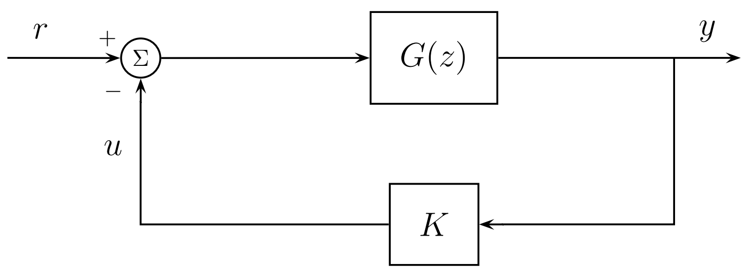

Figure 5: Closed-loop system.

i) Find the transfer function from r to y. [2p]

ii) Find the values of K for which the closed-loop transfer function is stable. [3p]

iii) If the feedback control system is given by $u[k] = Ky[k-1]$, find the values of K for which the closed-loop transfer function is stable. [4p]

(Turn over)

4. a) Consider the discrete-time process

$$x[k+1] = \begin{bmatrix} 0.9 & 0 \\ 1 & 0.7 \end{bmatrix} x[k] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[k]$$

$$y[k] = \begin{bmatrix} 0 & 1 \end{bmatrix} x[k]$$

i) Is the system reachable? What does this imply? [2p]

ii) Determine a state deadbeat controller that gives *unit* static gain, that is, determine L_c and L in the controller [2p]

$$u[k] = L_c y_{\text{ref}}[k] - Lx[k]$$

iii) Determine the stability range for the parameters in L, that is, use the controller from ii) and determine how much the other parameters may change before the closed-loop system becomes unstable. [3p]

b) Consider the discrete-time process

$$x[k+1] = \begin{bmatrix} 0.25 & 0.5 \\ 1 & 2 \end{bmatrix} x[k] + \begin{bmatrix} 1 \\ 4 \end{bmatrix} u[k]$$

$$y[k] = \begin{bmatrix} 1 & 0 \end{bmatrix} x[k]$$

i) Determine the state-feedback controller

$$u[k] = L_c y_{\text{ref}}[k] - Lx[k]$$

such that the states are brought to the origin in just two sampling intervals. [2p]

ii) Is it possible to find a state-feedback controller that can take the system from the origin to $x[k] = \begin{bmatrix} 2 & 8 \end{bmatrix}^T$? Justify your answer. [3p]

iii) Find an observer that estimates the state such that the estimation error, $\tilde{x}[k]$, decreases as $0.2^k \tilde{x}[k]$. [3p]