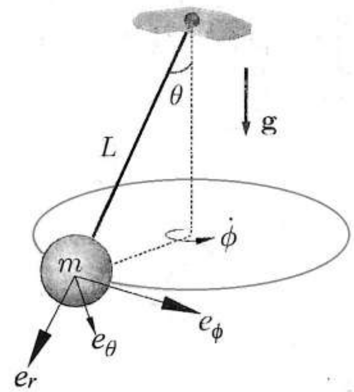
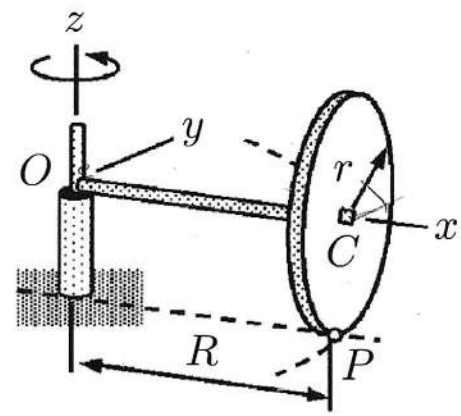
**MEC-E1010; Dynamics of Rigid Body, Exam** **19.10.2022**

**Note: this exam has two pages and there are five problems (max 30 points).**

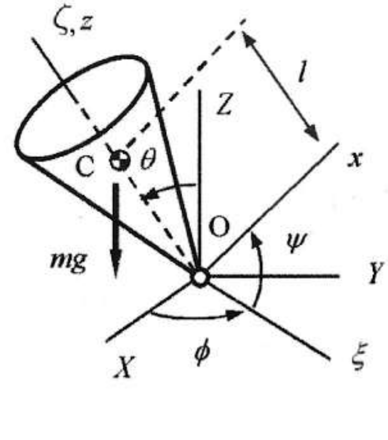
**Problem 1.** The sphere shown in the figure moves on a circular path so that the angle $\theta$ and angular velocity $\dot{\phi}$ remain constants. Determine the rates of change of base vectors $\mathbf{e}_r$, $\mathbf{e}_\phi$, and $\mathbf{e}_\theta$ of the spherical coordinate system for this case. Distance of the sphere from the attachment point, $L$, remains constant. **(6 p.)**



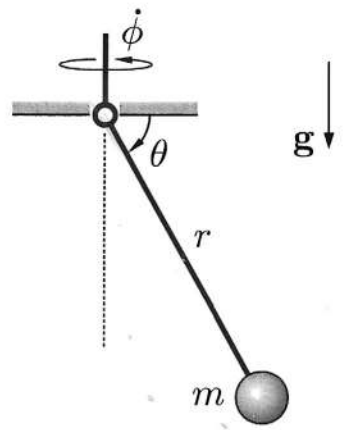

**Problem 2.** System in the figure consists of a thin disk (radius $r$) and a shaft (length $R$), which is hinged into point $O$ as shown in the figure. The disk rolls (with constant velocity) without slipping so that one full cycle of its point $C$ around the $z$-axis takes time $T$. Solve the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the disk. Use the equations of the relative motion of a rigid body. Give the solutions in the $xyz$-coordinate system (base $\mathbf{ijk}$) attached to the shaft (z axis always remains vertical as in the figure). **(6 p.)**




**Problem 3.** Derive the equations of motion for the conical spinning top of the figure in, 3-1-3 Eulerian angles related $\xi\eta\zeta$-frame (the tip of spinning top aligns always with point origin $O$ of the $\xi\eta\zeta$-frame). Start from conservation law $\mathbf{m} = \dot{\mathbf{l}}$. Components of the diagonal moment of inertia tensor are $I_{\xi\xi} = I_{\eta\eta} = I_0$ and $I_{\zeta\zeta} = I$. **(6 p.)**



**Problem 4.** Find the kinetic energy $T$ and the generalized force $Q$ of the rotating pendulum of the figure. Using those, derive the equations of motion of the system. Assume that the distance $r$ is constant, the angular velocity of the pendulum around its vertical axis $\dot{\phi}$ is constant, and that the mass $m$ of the system is concentrated on a single point. Use $\theta = \theta(t)$ as the generalized coordinate **(6 p.)**




**Problem 5.** The system of the figure consists of three masses ($M_1$, $M_2$, and $M_3$) and four linear springs (spring constants $k_0$, $k_1$, $k_2$, and $k_3$). Masses can only move into horizontal direction and there is no energy dissipation in the system. Derive the Lagrangian $L = T - V$ for the system, and use the Lagrangian $L$ to further derive the equations of motion of the system. **(6 p.)**