1. (a) The *divergence theorem* states that the volume integral of the divergence of a vector field equals the total outward flux of the vector field through the surface that bounds the volume.

   (b) *Lenz's law* says that the induced current due to an induced electromotive force creates a secondary magnetic field that opposes the change in magnetic flux. (The direction of any electromagnetic induction phenomenon oppose the original change.)

   (c) The *polarization* of a plane wave is defined by looking at the time dependent behavior of the electric field vector at a fixed position. The wave is circularly polarized if the tip of the vector draws a circle.

   (d) Total reflection happens at the planar interface between two media when the incident angle is larger that the *critical angle*. This angle exists for lossless media when the refractive index in medium 1 (where the incident wave propagates) is larger than the refractive index in medium 2.

2. Taking he curl of both sides of Faraday's law gives

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right) \quad \Leftrightarrow \quad \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{B}) .$$

If we assume an *homogeneous ordinary medium*, we have $\mathbf{D} = \varepsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$ with constant ε and μ. If we also assume that the *charge density is zero*, Gauss's law $\nabla \cdot (\varepsilon \mathbf{E}) = \rho = 0$ implies that $\nabla \cdot \mathbf{E} = 0$. After these assumptions we have

$$\nabla^2 \mathbf{E} = \mu \frac{\partial}{\partial t} (\nabla \times \mathbf{H}) .$$

Now we can use Ampère's law and assume that the *current density is zero* to get the wanted result

$$\nabla^2 \mathbf{E} = \mu \frac{\partial}{\partial t} \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \Rightarrow \quad \nabla^2 \mathbf{E} - \frac{1}{u^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mathbf{0}, \quad u = \frac{1}{\sqrt{\varepsilon\mu}}.$$

3. (a) The reflection and transmission coefficients are

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0/2 - \eta_0}{\eta_0/2 + \eta_0} = -\frac{1}{3}, \qquad \tau = 1 + \Gamma = \frac{2}{3}.$$

   (b) The total field in medium 1 is

$$\begin{aligned} \mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_r = \mathbf{a}_y E_0 e^{-j\beta_1 z} + \mathbf{a}_y \Gamma E_0 e^{+j\beta_1 z} &= \mathbf{a}_y E_0 \left(e^{-j\beta_1 z} - \frac{1}{3} e^{+j\beta_1 z}\right) \\ &= \mathbf{a}_y E_0 \left(\frac{2}{3} e^{-j\beta_1 z} - j\frac{2}{3}\sin(\beta_1 z)\right) \\ &= \mathbf{a}_y E_0 e^{-j\beta_1 z}\left(1 - \frac{1}{3} e^{+j2\beta_1 z}\right). \end{aligned}$$

   (Any one of the last three expressions is fine as the final answer.)

   (c) The standing-wave ratio in medium 1 is

$$S = \frac{|E|_{\max}}{|E|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.$$