Course Name: Production Planning and Optimization

Course Code: CHEM-E7151

Exam, 19.10.2022, 09.00-13.00

Exam Method: Written and MyCourses Assignment activity for file attachments.

Write your full name and student number on the top of all answer sheets.

Submit all files related to your answers to the Submission folder named "Exam 2022".

All questions need to be answered (total of 20 points).

Question 1. *(2 points)*

A paper company tries to find the production strategy to maximize its weekly profit. The company operates both a pulp mill, as well as a paper machine. The pulp mill is capable of producing 30 tons pulp per hour with the production cost of 500 EUR/ton, which can be sold for 800 EUR/ton. Alternatively, the company can use the produced pulp for paper production in the paper machine for a cost of 650 EUR/ton of produced paper. Note that since pulp contains lot of water, 80% of its weight evaporates in the drying section of the paper machine. The sales price for paper is 1300 EUR/ton and in practice the paper machine has an hourly capacity of 25 tons. The company can store maximally 500 tons of pulp and we assume that the weekly production is delivered one time at the end of the week.

a) Briefly answer:

- What type of a mathematical problem is this? (0.25 points)
- Which approach/es can be used to solve this problem? (0.25 points)

b) Formulate the given problem (also explain your model). **Do not solve it!** (1.5 points)

Question 2. *(4 points)*

Solve the following optimization problem with respect to variables $x_i, i \in \{1,2,3\}$ using the **simplex** method:

$$\max Z = x_1 + 2x_2 + 3x_3$$

$$\text{subject to } \; x_1 + x_2 + x_3 \leq 10$$

$$x_1 + x_2 - x_3 \geq -5$$

$$0 \leq x_1, x_2$$

$$0 \leq x_3 \leq 8$$

Question 3. *(3 points)*

A chemical company, based in Kajaani, is looking for a new route to transport their products to a customer in Dortmund, Germany. They have received offers from logistics companies, operating cargo ships and rail networks. The costs associated with each transportation segment are shown in Table 1. The elements marked by a dash indicate a segment, for which no offers were received or are not applicable. Use **Dynamic Programming** to determine the cheapest transportation route from Kajaani to Dortmund.

Hint: Start by identifying stages of the problem.

Table 1.

Segment cost [kEur/ton]		Destination						
		Kajaani	**Oulu**	**Helsinki**	**Rotterdam**	**Hamburg**	**Bremen**	**Dortmund**
Origin	**Kajaani**	-	1.5	2.5	-	-	-	-
	Oulu	-	-	-	4.5	3.5	5.0	-
	Helsinki	-	-	-	5.0	4.0	3.5	-
	Rotterdam	-	-	-	-	-	-	0.5
	Hamburg	-	-	-	-	-	-	2.0
	Bremen	-	-	-	-	-	-	1.0
	Dortmund	-	-	-	-	-	-	-

Question 4. (*5 points*)

A food company produces two readymade meals with fish (A) and meat (B) (the vegetarian option has been left out only to keep this problem two-dimensional). The production is both labor-intensive and consumes electricity due to the strict hygiene requirements (heating, pasteurization). Solve the problem of minimizing the daily production cost when each ton of A requires 2.5 hours of labor and 80 kWh of electricity, and each ton of B requires 1.75 hours of labor and 130 kWh of electricity. The plant has 10 employees (each working 8 hours per day), and an electricity contract limited to 1.4 MWh / day. The plant has one machine only, which **must operate continuously** – production times are 108 min/ton of meal A and 84 min for meal B. The labor cost is 25 EUR/hour and electricity cost (fixed) 8 EUR/kWh. The delivery of production must be done in exact tons (special distribution units with cooling) as the rest would have to be disposed.

a) **Formulate** the problem as an **integer programming** problem. (2 points)

b) **Solve** this model by using the **graphical** method **without integer restrictions** (relaxed LP problem). What special can you observe about this particular problem? (1 point)

c) Perform the **Branch and Bound** algorithm (draw the search tree) and **compare** the obtained optimum solution with the relaxed linear programming model solved in part b). You can solve the LP-steps either manually or using e.g. GAMS (in which case submit the code). (2 points)

Question 5 *(4 points)*

A process has two steps:

1. Equipment M1 takes raw material R1 in and after processing it for 1.5 hours, releases an intermediate I1. The capacity of M1 is 80 tons.
2. The second step has two alternative equipment. Equipment M2 consumes 35 tons of the intermediate product I1 and produces end-product P1 in 2 hours. Equipment M3 takes 50 tons of I1 and produces end-product P2 in 3 hours.

The initial raw material R1 amount is 200 tons, which is replenished by 100 tons every 4 hours.

In order to optimize the process:

a) Build the RTN graph (tasks, resources) and define the parameters $\mu_{i,r,\theta}$, $\nu_{i,r,\theta}$, δ, τ_i, $\pi_{r,t}$. (2 points)

b) Solve the problem in GAMS for one day (24 hours) maximizing the revenues when the price is 120 EUR/ton for product 1 and 150 EUR/ton for product 2. (2 points)

Question 6. ***(2 points)***

A company is considering to move the production of certain energy-intensive batch processes to nighttime (from midnight to 6 am) due to the increased electricity price. Typically, at most 10 batches need to be produced daily. These typical batches have the durations of 1, 1, 2, 2, 2, 3, 3, 4, 5, and 6 hours. All batches can be produced on the same type of a reactor. However, the reactor needs to be manually cleaned between processing two different batches. The cleaning takes an hour (the cleaning after the last batch process can be performed after 6 am). The company is investigating how many reactors they need for shifting the production to nighttime. Use the "First-fit decreasing" heuristic algorithm to allocate the typical batches to reactors. Visualize a schedule (using a Gantt chart) that could result from the allocation. How many reactors the company should have?