1. a) $Ab = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 4-6 \\ 12+3 \end{pmatrix} = \begin{pmatrix} -2 \\ 15 \end{pmatrix}$

b) $\begin{pmatrix} 4 & -3 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = -4-6 = -10$

c) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}\begin{pmatrix} -1 & 2 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 3 & -6 \end{pmatrix}$

d) Jos om. vektori, niin $Bd = \lambda d$

$$\underbrace{\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}}_{B}\underbrace{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}_{d} = \begin{pmatrix} -1+2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \underbrace{-1}_{\lambda}\underbrace{\begin{pmatrix} -1 \\ 1 \end{pmatrix}}_{d}$$

$\Rightarrow$ on om. vektori!

2. a)

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 1 & 2 & -1 & -3 \\ -2 & -3 & 0 & 3 \end{array}\right) \quad -1, \; +2$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 0 & -1 & -3 & -7 \\ 0 & 3 & 4 & 11 \end{array}\right) \quad +3$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 0 & -1 & -3 & -7 \\ 0 & 0 & -5 & -10 \end{array}\right)$$

$\Rightarrow -5x_3 = -10$

$x_3 = \frac{-10}{-5} = 2$

$-x_2 - 3 \cdot 2 = -7$

$-x_2 = -7 + 6 = -1$

$x_2 = 1$

$x_3 + 3 \cdot 1 + 2 \cdot 2 = 4$

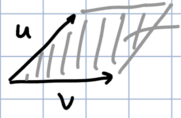
$x_3 = 4 - 4 - 3 = -3$

b) ovat (koska tukialkiot $\neq 0$)

3. a) $(2 \; -1 \; 0) \cdot (1 \; 1 \; 3) = 2 - 1 = 1 \neq 0$

eivät ole kohtisuorassa, sillä sisätulo $\neq 0$

b)



suunnikkaan ala :

$$|u \times v| = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 1 & 1 & 3 \end{vmatrix} = |i(-3-0) + j(6-0) + k(2-(-1))|$$

$$= |-3i + 6j + 3k|$$

$$= \sqrt{3^2 + 6^2 + 3^2} = \sqrt{9+36+9} = \sqrt{54}$$

$$= 3\sqrt{6}$$

$\Rightarrow$ kolmion ala on puolet suunnikkaan alasta:

Vastaus: $\dfrac{3\sqrt{6}}{2}$

4.

a) $$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \end{bmatrix}$$

b) $$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

c) $$A^4 = A^3 \cdot A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^{k+4} = A^k \underbrace{A^4}_{0} = A^k \cdot 0 = 0 \quad \forall k \geq 1$$

$\Rightarrow$ erityisesti $A^{2022} = A^{2018+4} = A^{2018} \cdot \underbrace{A^4}_{0} = 0$

VASTAUS: $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$