ELEC-E7210 Communication Theory

This is an open book exam. All material printed or written on paper allowed. No electronic equipment except scientific calculator. The exam can be written in Finnish, Swedish or English.

Exam 8.12. 2022

1. Answer the questions below shortly, with at most a couple of sentences.

   a) Consider a coding and modulation scheme where a binary rate 2/3 code is combined with QPSK modulation. Give a simple upper bound for the throughput of this scheme.

   b) Consider a real discrete time AWGN channel with SNR 3. The transmitter is using BPSK modulation. Is it possible to achieve throughput 2/3 (bits/channel use) with this modulation in this channel? Explain your result.

   c) Transmitter and receiver are connected with two identical but independent parallel AWGN channels. What is the maximal multiplexing gain in this channel model?

   d) Consider a discrete time Rayleigh i.i.d block fading channel with average SNR 1 and perfect CSI at the transmitter. The transmitter has a maximum power constraint, such that power cannot be allocated across time. Give an upper bound for the capacity in this channel.

   e) Why is the capacity often not a relevant performance measure in block fading channels without CSI at transmitter?

2. The mobile radio channel can be described by the coherence time, and the coherence bandwidth. Answer the questions below shortly, with at most a couple of sentences.

   a) What is the relation of the coherence bandwidth to the delay spread of the channel?

   b) Using the coherence bandwidth and the system bandwidth, how do you define a wideband (frequency selective) vs. narrow band (frequency flat) fading channel?

   c) Which technical solutions are needed when communicating over a frequency selective (but not frequency flat) channel?

   d) What is the relation of the coherence time to the Doppler spread of the channel, and what is the relation of Doppler spread to speed?

   e) Using the coherence time and the symbol period, how do you define a slowly vs. rapidly fading channel?

   f) Which solutions can be used when communicating over slow (but not rapid) fading channels?

3. Assume a time-domain block transmission in a tapped delay line channel with two channel taps $[h_0 \;\; h_1] = [3 \;\; 1]$. Each transmission block of two symbols is prepended with a cyclic prefix of length one. The symbols transmitted in discrete time instances $[-1, \ldots, 2]$ are thus $[x_0 \;\; x_2 \;\; x_1 \;\; x_2]^{\mathrm{T}}$, where x_1 and x_2 are the block of interest.

   a) Construct the Toeplitz channel matrix that gives the received samples at discrete time instances $[0, 1, 2]$ when multiplying a vector of transmitted signals, assuming perfect timing.

b) Considering the received sample vector $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ in discrete time instances $[1, 2]$, construct the circulant channel matrix $\mathbf{H}$ that describes the received signals in a signal model $\mathbf{y} = \mathbf{Hx} + \mathbf{n}$, where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the block of interest and $\mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$ is additive noise.

c) The 2x2 DFT matrix is $\mathbf{M} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. To start with frequency domain equalization, diagonalize the circulant channel matrix $\mathbf{H}$ using the DFT matrix.

d) You want to transmit the QPSK symbols $s_1 = \frac{1}{\sqrt{2}}(1 + i)$ and $s_2 = \frac{1}{\sqrt{2}}(1 - i)$ during time instances $[1, 2]$ using block size $N = 2$ OFDM. What are the transmitted time domain signals $[x_1, x_2]$?

4. Consider a channel with M receiver antenna diversity branches and i.i.d. Rayleigh fading on each branch, with white complex Gaussian noise. The instantaneous SNR γ of each of the M independent diversity branches comes from the PDF

$$f(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}$$

where $\overline{\gamma}$ is the average SNR, same for all branches.

a) Let us assume that $M = 1$ and that we are using BPSK and targeting bit error probability 10^{-3} or less. If the average SNR $\overline{\gamma}$ is 10 dB what is the outage probability?

b) Assume now that $M = 2$, $\overline{\gamma}$ is again 10 dB. We use selection combining, i.e. the system selects the branch with the highest SNR. What is the outage probability?

Note that for BPSK in AWGN channel the bit error probability can be approximated by $P_b \approx Q(\sqrt{2\gamma})$, where γ is the instantaneous SNR. We also have that $Q(3) = 0.00135, Q(3.1) = 0.00097$ and $Q(2.8) = 0.00256$.

5. We have a static MIMO channel with 2 Tx and 2 Rx antennas:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{1}$$

where the transmitted symbols x_1 and x_2 are independent with expected symbol energy $E[x_i^* x_i] = 1$ and noise symbols n_1 and n_2 are independent complex Gaussian with zero mean and variance $E[n_i^* n_i] = 1$.

a) Assume that the receiver uses zero-forcing. What is the post-processing Signal-to-Interference-and-Noise (SINR) experienced by symbols x_1 and x_2? Hint: Note that noise affecting transmitted symbols x_1 and x_2 can be amplified with different factors in post-processing.

b) The same question, but assuming that the channel matrix would be $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.