MEC-E1020 Fluid dynamics - Exam - 20.10.2022

Remember to write your name on your "cheat sheets" and to return them with the exam papers.

1 Fundamental equations and their solutions

Let's consider a steady, fully developed, two-dimensional, laminar and incompressible flow between two parallel plates, which are separated by a distance $h = 1\,\text{mm}$. Both plates are stationary. The pressure is decreasing in the direction of the flow with $dp/dx = -23\,\text{kPa/m}$. The fluid between the plates has a density $\rho = 817\,\text{kg/m}^3$ and a dynamic viscosity $\mu = 19.2 \times 10^{-3}\,\text{kg/(m s)}$.

a) What can you say about the acceleration of a fluid particle in this case? Justify your answer. (1p)

b) By starting from the continuity and momentum equations show that in this case the net shear force has to be balanced by the net pressure force. When dropping out terms, justify this separately for each term. (2p)

c) Calculate the shear stress acting on the upper plate. (2p)

d) We need energy to keep the flow running. Discuss the balance of mechanical energy in this case. (1p)

2 Boundary layers and related flows

a) It can be shown that within a boundary layer some of the terms in the full Navier-Stokes equations can be neglected. How can this be justified? How does this change the way in which the equations can be solved? (2p)

b) Explain, why the boundary layer developing on the surface of a body is displacing the streamlines away from the body. How can you evaluate the size of this displacement, if you know the velocity profile within the boundary layer? (2p)

c) It is assumed that the pressure does not vary within the boundary layer in the wall-normal direction. However, pressure can vary in the direction of the flow and transversal to the flow. Discuss the influence of these (i.e. longitudinal and transversal) pressure variations on the boundary layer flow. (2p)

3 Instability and turbulence

a) Explain, what we mean by the energy cascade and what is the role of the viscosity in this? (2p)

b) Explain, what the eddy viscosity hypothesis means. (1p)

c) Let's assume that the characteristics of a turbulent flat plate boundary layer are given by

$$\delta \approx \frac{0.16x}{\mathrm{Re}_x^{1/7}} \qquad c_f \approx \frac{0.027}{\mathrm{Re}_x^{1/7}} ,$$

where δ is the boundary layer thickness, $c_f = 2\tau_0/(\rho U_\infty^2)$ is the nondimensional shear stress on the wall and Re_x is the local Reynolds number based on the distance from the leading edge of the plate x. Study the boundary layer velocity profile at $x = 10\,\mathrm{m}$, when the freestream velocity U_∞ is $10\,\mathrm{m/s}$, the density of the fluid is $1000\,\mathrm{kg/m^3}$ and the kinematic viscosity is $1.0 \times 10^{-6}\,\mathrm{m^2/s}$. Evaluate in which region of the boundary layer the point with a distance of $0.40\,\mathrm{mm}$ from the wall is located, and evaluate the dimensional velocity U at this point. (3p)

4 Numerical techniques

a) Explain what is the difference between verification and validation. (1p)

b) Describe the different types of error in numerical predictions and the origins of the errors. (2p)

c) A discretised form of the one-dimensional convection-diffusion equation is given as

$$T_i^n + \alpha\left(T_i^n - T_{i-1}^n\right) - \beta\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) = T_i^{n-1}$$

$$\alpha = \frac{u\Delta t}{\Delta x} , \qquad \beta = \frac{D\Delta t}{\Delta x^2} ,$$

where u is assumed to be a positive velocity and D a diffusion coefficient. The subscripts refer to different points of the spatial discretisation and the superscripts to the discrete time levels. Calculate the truncation error in this case. What is the spatial and temporal order of accuracy of the scheme? (3p)