

Question 1: Explain the following concepts (2p). Use mathematical notation when needed.

- a) Value-at-Risk as a risk measure **(0.5p)**
- b) Certainty equivalent of a random lottery X **(0.5p)**
- c) Approval voting **(0.5p)**
- d) Representativeness bias **(0.5p)**

Question 2: Are the following statements true or false (2p)? Justify your answers.

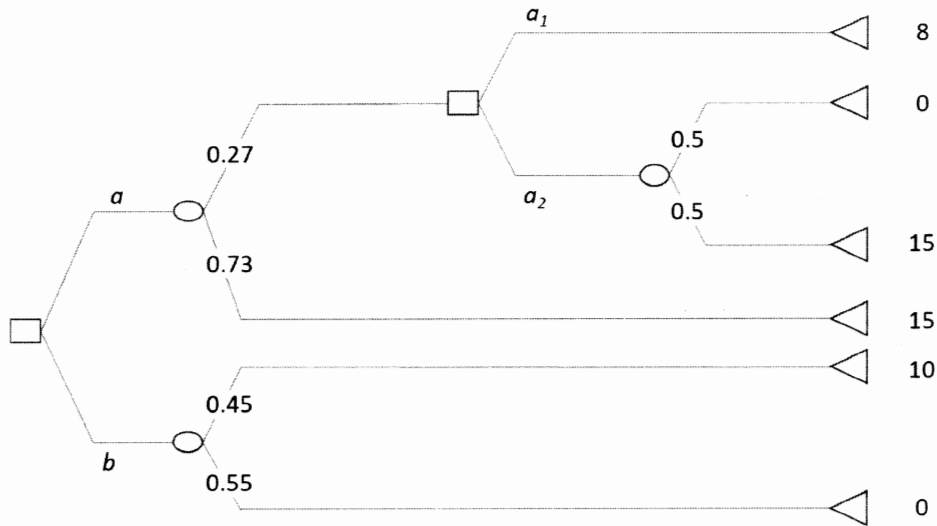
- a) An additive overall value function assumes that the attributes are additive independent. **(0.5p)**
- b) Adding a new alternative to an AHP model cannot change the relative priority ranking of the original alternatives. **(0.5p)**
- c) A utopian point in multi-objective optimization is a point at which there is at least one objective function that has a strictly better value than that for any feasible solution. **(0.5p)**
- d) In the EUT, any DM with a convex (but not linear) utility function is risk seeking. **(0.5p)**

Question 3: Multi-attribute Value Theory (MAVT) (4p). Consider a DM is choosing an apartment. The attributes are a_1 : *area* (between 25 and 50m²) and a_2 : *rent* (between 500 and 900 €/month). Your task is to construct an additive multi-attribute value function that is consistent with the DM's preferences:

1. The value of *area* is increasing, $(35 \leftarrow 25) \sim_d (50 \leftarrow 35)$ and $(45 \leftarrow 35) \sim_d (50 \leftarrow 45)$. You can assume that within the intervals [25,35], [35,45] and [45,50] the value function is linear.
 2. The value of *rent* is decreasing and linear.
 3. Apartments (25,500) and (40,700) are equally preferred.
- a) Construct normalized attribute-specific value functions v_i^N which map the worst levels to zero and best level to one. **(1p)**
 - b) Derive attribute weights for the additive value function $V(x) = \sum_{i=1}^2 w_i v_i^N(x_i)$. **(1p)**
 - c) Compute overall values for the apartments, $x^1 = (50,900)$, $x^2 = (25,500)$ and $x^3 = (35,700)$. **(1p)**
 - d) Which of the three apartments x^1 , x^2 , and x^3 are non-dominated when the last preference statement about the equality of apartments (25,500) and (40,700) is changed to the statement that apartment (25,600) is preferred to (50,900)? **(1p)**

Justify your answers. Graphical presentation is an adequate justification for d).

Question 4: Utility theory (4p). The following figure presents a decision tree with three strategies (i.e., $a - a_1 ; a - a_2 ; b$) to obtain the cash flows indicated in the leaf nodes on the right of the tree. Nick and Cyndia seek to maximize their expected cash flow in accordance with the axioms of utility theory. **Justify your answers to questions a)–d).**



- Which of the three strategies are dominated in the sense of first order stochastic dominance (FSD)? **(1 p)**
- Which of the three strategies are dominated in the sense of second order stochastic dominance (SSD)? **(1 p)**
- For any $c > 0$, Nick is indifferent between getting the sure payment c and any risky cash flow C with $E(C) = c$. Which of the 3 strategies would Nick choose? What is Nick's risk attitude? **(1p)**
- Cyndia's utility function is $U(x) = kx^2$ for some $k > 0$, for all non-negative x . Which strategy would Cyndia choose? What is Cyndia's risk attitude? **(1p)**

Question 5: MAVT (4p). The DM is considering between four job opportunities with the help of an additive multiattribute value model. The data for this model is shown in the spreadsheet below.

		Attribute weights						
		Salary	Vacation	Fit with interests				
		0.5	0.25	0.25				
		Attribute-specific performances			Attribute-specific values			Overall value
		Salary (€)	Vacation (weeks)	Fit with interests	Salary	Vacation	Fit with interests	$V(x_1, x_2, x_3)$
Job	x_1	x_2	x_3	$v_1(x_1)$	$v_2(x_2)$	$v_3(x_3)$	$V(x_1, x_2, x_3)$	
x^A0	2500	1	poor	0.00	0.00	0.00	0.00	
x^{A*}	5500	5	excellent	1.00	1.00	1.00	1.00	
x^A	4300	3	poor	0.77	0.50	0.00	0.51	
x^B	4750	5	fair	0.87	1.00	0.40	0.79	
x^C	3250	2	good	0.50	0.25	0.60	0.46	
x^D	3700	4	excellent	0.63	0.75	1.00	0.75	

- a) When assessing the attribute-specific value functions, the DM was asked to assess the number of vacation weeks x_2 such that she would be indifferent between changes $(4300, x_2, \text{excellent}) \leftarrow (4300, 1, \text{excellent})$ and changes $(4300, 5, \text{excellent}) \leftarrow (4300, 4, \text{excellent})$. What was her response for the value of x_2 ? Please explain why. (1p)
- b) When assessing the weights for the model, the DM was asked to determine salary x_1 such that she would be indifferent between jobs $(x_1, 1, \text{good})$ and $(2500, 5, \text{good})$. What was her response for the value of x_1 ? Please explain why. (1p)

Next, assume that the DM has given only incomplete information about the attribute weights so that the feasible weights are constrained with the equation $w_2 = w_3$. This leads to the interval-valued overall values for the four jobs as shown below.

- c) Which one(s) of the four jobs are non-dominated? (1p)
- d) The DM gives an additional statement that job $(5500, 3, \text{poor})$ is preferred to job $(2500, 3, \text{excellent})$. Which job(s) would be recommended by the MAVT model? Please justify. (1p)

