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**A”**Matrix Algebra  
MS-A0001  
Hakula/Orlich  
Course Exam, 13.12.2022**T**

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**This exam is for those participating on continuous assessment.** Every question carries an equal weight, similarly every part of a question carries an equal weight, unless otherwise specified. There are six problems on this exam. Calculators are not permitted.

**PROBLEM 1** Let

$$v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \end{pmatrix}, \quad u = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

Compute the following products or indicate if they are not defined. (a)  $Au$ , (b)  $Bv$ , (c)  $u^T A$ , (d)  $v^T B$ , (e)  $u^T Au$ , (f)  $v^T BA$ .

**PROBLEM 2** Solve the linear system

$$\begin{cases} \sqrt{\frac{2}{3}}\xi_1 + \frac{1}{2}\xi_2 - \frac{1}{2\sqrt{3}}\xi_3 = 1 \\ \frac{1}{\sqrt{3}}\xi_1 - \frac{1}{\sqrt{2}}\xi_2 + \frac{1}{\sqrt{6}}\xi_3 = 2 \\ \frac{1}{2}\xi_2 - \frac{\sqrt{3}}{2}\xi_3 = 3 \end{cases}.$$

**PROBLEM 3** Let

$$A = \begin{pmatrix} 1 & 3 & 8 \\ 1 & 2 & 6 \\ 0 & 1 & 2 \end{pmatrix}.$$

Find the nullspace of  $A^T A$ , that is,  $N(A^T A)$ .

**PROBLEM 4** Let the product  $PA$  be

$$PA = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 2 & -1 & 4 \\ 6 & -2 & 9 \end{pmatrix}.$$

Find the  $LU$ -decomposition  $PA = LU$ .

**PROBLEM 5** Consider the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \end{pmatrix}.$$

- (a) What is the rank of  $A$ ?
- (b) Compute  $\det(A^T A)$ .
- (c) Compute  $\det(AA^T)$ .

**PROBLEM 6** Let  $A$  be an invertible square matrix such that  $(\lambda, x)$  is an eigenpair,  $\lambda \neq 0$ ,  $x \neq o$ . Show that  $(1/(\lambda^k), x)$  is an eigenpair of  $A^{-k}$ , where  $k \geq 1$ ,  $k \in \mathbb{N}$ .