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PHYS-E0460 Introduction to Reactor Physics, 2nd mid-term exam 15 Dec 2022

You may use an unprogrammed calculator and the document "Mathematical Tools for Reactor Physics". You are welcome to answer in English, Finnish or Swedish.

1. Give a concise explanation of the following:
   a) point kinetics b) dollar (\$) c) void coefficient α_v
   d) Fourier's law e) hot channel factor F f) diversity.

2. A uranium-fuelled thermal reactor is operating at a constant thermal power $P_0 = 200$ MW. Control rods are pulled outwards from the core, inserting a positive reactivity $\rho = 0.0005$. Calculate the time required to reach the power $P = 300$ MW neglecting reactivity feedback effects. Take into account the prompt jump and compare the result to the approximation where the power is assumed to increase at stable period from its initial value. For simplicity, assume just one effective delayed neutron precursor group with a decay constant $\lambda = 0.1\ \mathrm{s}^{-1}$. For thermal reactors, the prompt neutron lifetime $l_p \approx 0.0001$ s, and for ^{235}U $\beta = 0.0065$. Power density is directly proportional to flux, which during the prompt jump behaves as

$$\phi_T = \phi_{T0} e^{-t/T} + \frac{\beta \phi_{T0}}{1-(1-\beta)k_\infty}(1-e^{-t/T}), \text{where } T = \left| \frac{l_p}{(1-\beta)k_\infty - 1} \right| .$$

   The stable period can be solved from the reactivity equation

$$(1-\beta)k_\infty + \frac{\lambda \beta k_\infty}{\omega + \lambda} - 1 = \omega l_p .$$

3. The thermal absorption cross section of the reactor poison ^{157}Gd is $\overline{\sigma}_{aG} = 2.5 \times 10^5$ barn. ^{157}Gd is stable and born in the β^- decay chain: $\cdots \xrightarrow{\beta^-} {}^{157}\mathrm{Sm} \xrightarrow{\beta^-} {}^{157}\mathrm{Eu} \xrightarrow{\beta^-} {}^{157}\mathrm{Gd}$(stable). ^{157}Sm has a half-life of 8 min and its yield is 6×10^{-5} atoms per ^{235}U fission. ^{157}Eu has a half-life of 15.2 h and its yield directly from fission is very small. Let's assume a pure ^{235}U fuel so that $p = \epsilon = 1$, $\nu = 2.42$ and $\beta = 0.0065$.
   a) Derive the expression for the reactivity effect of a reactor poison (hint: only fuel utilization f is affected in the multiplication factor k).
   b) What is the reactivity effect of ^{157}Gd in equilibrium, if the average thermal flux of the reactor is $\phi_T = 2.5 \times 10^{14}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$?
   c) If the reactor has been running for a long time at the mentioned average flux, what is the maximum reactivity effect of ^{157}Gd after shutdown?

4. Consider a coolant channel of the centermost fuel rod in a PWR core of height H. The density of the coolant is ρ, its specific heat is c_p and its flow velocity is v. The

cross-section areas of the channel and the fuel in the rod are A_c and A_f. The coolant inlet temperature is T_{b0}, the convective heat transfer coefficient from rod surface to coolant is h, and the axial distribution of power is of cosine form. Determine the bulk temperature of the coolant $T_b(z)$ and the outer surface temperature $T_c(z)$ of the rod cladding as functions of the axial coordinate z (along the height of the core).

5. Describe the goals and means of fuel management.

Mathematical Tools for Reactor Physics

1 Delta function

Definition

$$\delta(\mathbf{r}-\mathbf{r}_0) = 0, \quad \text{if} \quad \mathbf{r} \neq \mathbf{r}_0$$

$$\int_V \delta(\mathbf{r}-\mathbf{r}_0)\, dV = \begin{cases} 1, & \text{if} \quad \mathbf{r}_0 \in V \\ 0, & \text{if} \quad \mathbf{r}_0 \notin V \end{cases}$$

Consequence

$$\int_V \delta(\mathbf{r}-\mathbf{r}_0) f(\mathbf{r})\, dV = f(\mathbf{r}_0) \int_V \delta(\mathbf{r}-\mathbf{r}_0)\, dV = \begin{cases} f(\mathbf{r}_0), & \text{if} \quad \mathbf{r}_0 \in V \\ 0, & \text{if} \quad \mathbf{r}_0 \notin V \end{cases}$$

δ in different coordinates

- Rectangular coordinates: $\delta(\mathbf{r}-\mathbf{r}_0) = \delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$
- Cylinder coordinates: $\delta(\mathbf{r}-\mathbf{r}_0) = \frac{1}{r}\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)$
- Spherical coordinates: $\delta(\mathbf{r}-\mathbf{r}_0) = \frac{1}{r^2 \sin\theta}\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(\theta-\theta_0)$

Singular sources in terms of the delta function

- Point source at $\mathbf{r}_0$: $S(\mathbf{r}) = S\delta(\mathbf{r}-\mathbf{r}_0)$
- Plane symmetry
  - Plane source at $x = x_0$: $S(\mathbf{r}) = S\delta(x-x_0)$
- Cylinder symmetry
  - Thin, hollow cylindrical source of radius $\mathbf{r}_0$: $S(\mathbf{r}) = \frac{S}{2\pi r}\delta(r-r_0)$
  - Line source on z axis: $S(\mathbf{r}) = \frac{S}{2\pi r}\delta(r)$
- Spherical symmetry
  - Thin, hollow spherical source of radius $\mathbf{r}_0$: $S(\mathbf{r}) = \frac{S}{4\pi r^2}\delta(r-r_0)$
  - Point source at the origin: $S(\mathbf{r}) = \frac{S}{4\pi r^2}\delta(r)$

2 Bessel functions

J_n and Y_n are Bessel functions, I_n and K_n modifioied Bessel functions. J_n and Y_n have infinite number of zeros, I_n increases monotonically, and K_n decreases monotonically. At the origin and infinity the functions behave as follows:

$$J_0(0) = 1, \quad J_n(0) = 0 \quad (n \neq 0), \quad \lim_{x \to 0} Y_n(x) = -\infty$$

$$I_0(0) = 1, \quad I_n(0) = 0 \quad (n \neq 0), \quad \lim_{x \to 0} K_n(x) = \infty$$

$$\lim_{x \to \infty} I_n(x) = \infty, \quad \lim_{x \to \infty} K_n(x) = 0.$$

For derivatives:

$$\frac{d}{dx}\left(x^n K_n(x)\right) = -x^n K_{n-1}(x), \quad \text{for others} \quad \frac{d}{dx}\left(x^n Z_n(x)\right) = x^n Z_{n-1}(x)$$

$$I_0'(x) = I_1(x), \quad \text{for others} \quad Z_0'(x) = -Z_1(x)$$

$$I_n(x)K_{n-1}(x) + I_{n-1}(x)K_n(x) = \frac{1}{x}$$

3 Integral theorems

$$\oint_{\partial V} \mathbf{F} \cdot d\mathbf{S} = \int_V (\nabla \cdot \mathbf{F})\, dV \qquad \text{(Gauss theorem)}$$

$$\oint_{\partial S} \mathbf{F} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} \qquad \text{(Stokes theorem)}$$

4 Coordinate systems and differential operators

Rectangular coordinates $(x,\ y,\ z)$

$$\nabla u = \frac{\partial u}{\partial x}\mathbf{e}_x + \frac{\partial u}{\partial y}\mathbf{e}_y + \frac{\partial u}{\partial z}\mathbf{e}_z$$

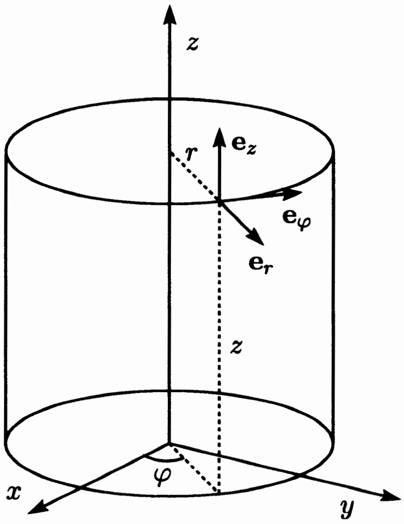
$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{e}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\mathbf{e}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{e}_z$$

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Cylindrical coordinates (r, φ, z)

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ z = z \end{cases}$$



$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \varphi = \arctan \frac{y}{x} \\ z = z \end{cases}$$

$$\begin{cases} \mathbf{e}_x = \mathbf{e}_r \cos\varphi - \mathbf{e}_\varphi \sin\varphi \\ \mathbf{e}_y = \mathbf{e}_r \sin\varphi + \mathbf{e}_\varphi \cos\varphi \\ \mathbf{e}_z = \mathbf{e}_z \end{cases}$$

$$\begin{cases} \mathbf{e}_r = \mathbf{e}_x \cos\varphi + \mathbf{e}_y \sin\varphi \\ \mathbf{e}_\varphi = -\mathbf{e}_x \sin\varphi + \mathbf{e}_y \cos\varphi \\ \mathbf{e}_z = \mathbf{e}_z \end{cases}$$

$$\nabla u = \frac{\partial u}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial u}{\partial \varphi}\mathbf{e}_\varphi + \frac{\partial u}{\partial z}\mathbf{e}_z$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r}\frac{\partial (rF_r)}{\partial r} + \frac{1}{r}\frac{\partial F_\varphi}{\partial \varphi} + \frac{\partial F_z}{\partial z}$$

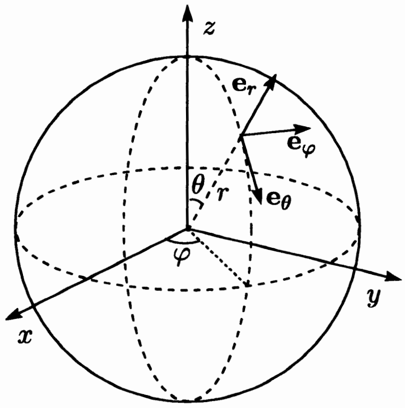
$$\nabla \times \mathbf{F} = \left(\frac{1}{r}\frac{\partial F_z}{\partial \varphi} - \frac{\partial F_\varphi}{\partial z}\right)\mathbf{e}_r + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\mathbf{e}_\varphi + \frac{1}{r}\left(\frac{\partial (rF_\varphi)}{\partial r} - \frac{\partial F_r}{\partial \varphi}\right)\mathbf{e}_z$$

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial z^2}$$

Spherical coordinates $(r,\ \theta,\ \varphi)$

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \arccos\frac{z}{\sqrt{x^2+y^2+z^2}} \\ \varphi = \arctan\frac{y}{x} \end{cases}$$



$$\begin{cases} \mathbf{e}_x = \mathbf{e}_r\sin\theta\cos\varphi + \mathbf{e}_\theta\cos\theta\cos\varphi - \mathbf{e}_\varphi\sin\varphi \\ \mathbf{e}_y = \mathbf{e}_r\sin\theta\sin\varphi + \mathbf{e}_\theta\cos\theta\sin\varphi + \mathbf{e}_\varphi\cos\varphi \\ \mathbf{e}_z = \mathbf{e}_r\cos\theta - \mathbf{e}_\theta\sin\theta \end{cases}$$

$$\begin{cases} \mathbf{e}_r = \mathbf{e}_x\sin\theta\cos\varphi + \mathbf{e}_y\sin\theta\sin\varphi + \mathbf{e}_z\cos\theta \\ \mathbf{e}_\theta = \mathbf{e}_x\cos\theta\cos\varphi + \mathbf{e}_y\cos\theta\sin\varphi - \mathbf{e}_z\sin\theta \\ \mathbf{e}_\varphi = -\mathbf{e}_x\sin\varphi + \mathbf{e}_y\cos\varphi \end{cases}$$

$$\nabla u = \frac{\partial u}{\partial r}\mathbf{e}_r + \frac{1}{r}\frac{\partial u}{\partial\theta}\mathbf{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial u}{\partial\varphi}\mathbf{e}_\varphi$$

$$\nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial(r^2F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(F_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}$$

$$\nabla\times\mathbf{F} = \frac{1}{r\sin\theta}\left(\frac{\partial(F_\varphi\sin\theta)}{\partial\theta} - \frac{\partial F_\theta}{\partial\varphi}\right)\mathbf{e}_r + \frac{1}{r\sin\theta}\left(\frac{\partial F_r}{\partial\varphi} - \sin\theta\frac{\partial(rF_\varphi)}{\partial r}\right)\mathbf{e}_\theta + \frac{1}{r}\left(\frac{\partial(rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right)\mathbf{e}_\varphi$$

$$\nabla^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\varphi^2}$$

References

L. Råde and B. Westergren, BETA – Mathematics Handbook for Science and Engineering, 3rd ed. Studentlitteratur, Lund, 1995.