Prediction and Time Series Analysis
Materials: In this exam you may have your pens and pencils, a ruler and an eraser. On top of that you may have one A4 of notes. The rules for the note are: size A4, text on one side only, it must be hand-written, your name has to be on the top right corner of the note.
Answer to all the questions.
In problems 1 and 5, you do not have to justify your answer. In all the other problems, justify your solutions and write down all your calculations.

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1. True or False (6 p.)

   Determine whether the statement is true or false. You do not have to justify your answers. Simply state wether the statement is true or false. (Every correct answer +1 p., every wrong answer -1 p., no answer 0 p.)

   (a) In the context of linear regression, the traditional least-squares estimators are sensitive to outlying observations.

   (b) In the context of linear regression, the variance inflation factor (VIF) is a measure of heteroscedasticity.

   (c) The autocorrelation function of a pure autoregressive process of order 3 is always equal to 0 after 3.

   (d) An autoregressive process of order $> 1$ is never stationary.

   (e) In the context of ARIMA-modeling, differencing can be applied in order to remove a linear trend.

   (f) In simple exponential smoothing, the value of $\hat{x}_{t+1|t}$ is predicted using a weighted sum of the previous observation $x_t, x_{t-1}, x_{t-2}, \ldots$

2. Linear regression (6 p.)

   Consider the linear regression model

   $$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i.$$

   You have a sample and you estimate the parameters $\beta_0$, $\beta_1$ and $\beta_2$ using traditional least squares estimators. You are worried about possible heteroscedasticity and you decide to apply the White homoscedasticity test.

   (a) Give the corresponding White test model and the White homoscedasticity test statistic. (2 p.)

(b) Give the null hypothesis of the test. (1 p.)

(c) Assume that the $\epsilon_i$ are not normally distributed. Explain, step by step, how you can estimate the $p$-value of the test statistic. (3 p.)

3. Stationarity (6 p.)

Let $x_t$ and $z_t$ be weakly stationary stochastic processes such that, for all $t, s \in \mathbb{Z}$, we have that

$$E[x_t] = 2,\ E[z_t] = 1,\ E[x_t^2] = 6,\ E[z_t^2] = 4,$$

$$E[(x_t - E[x_t])(x_s - E[x_s])] = \frac{2}{1 + |t - s|},$$

$$E[(z_t - E[z_t])(z_s - E[z_s])] = \frac{3}{1 + (t - s)^2},$$

and

$$E[(x_t - E[x_t])(z_s - E[z_s])] = 0.$$

Let $y_t = x_t + z_t$. Show that the process $y_t$ is weakly stationary.

4. Interval bootstrapping/block bootstrapping (6 p.)

Assume that you have observed a time series $x_1, x_2, x_3, ..., x_{13052}$. Based on plotting the series and its estimated autocorrelation and partial autocorrelation -functions, you think that the observed series is an MA(2) process

$$x_t = \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t, \quad (\epsilon_t)_{t \in T} \sim WN(0, \sigma^2)$$

and you have estimated the parameters $\theta_1$ and $\theta_2$. Your estimates are $\hat{\theta}_1 = 0.34$ and $\hat{\theta}_2 = -0.11$.

(a) Explain, step by step, how to construct a 90% bootstrap confidence interval for the parameter $\theta_2$. How do you construct a bootstrap sample when you have time series data? How many bootstrap samples do you take? What are the sample sizes of your bootstrap samples? How do you use the bootstrap samples in constructing the confidence interval? (4 p.)

(b) Your estimated 90% bootstrap confidence interval for $\theta_2$ is $(-0.2, -0.07)$. Based on that, does it seem that the parameter $\theta_2$ is statistically significant? (1 p.)

(c) Explain what is the main difference between bootstrapping when you have iid observations and bootstrapping when you have time series data. (1 p.)

5. Autocorrelations (6 p.)

Figures 1 and 2 display the theoretical autocorrelation and partial autocorrelation -functions of six different processes. Answer to the following questions. You do not have to justify your answers. (Every correct answer +1 p., every wrong answer 0 p., no answer 0 p.)

(a) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a MA(1)-process?

(b) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an MA(2)-process?

(c) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(3)-process?

(d) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an ARMA(2,2)-process?

(e) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SMA(3)_3$-process?

(f) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SAR(2)_3$-process?

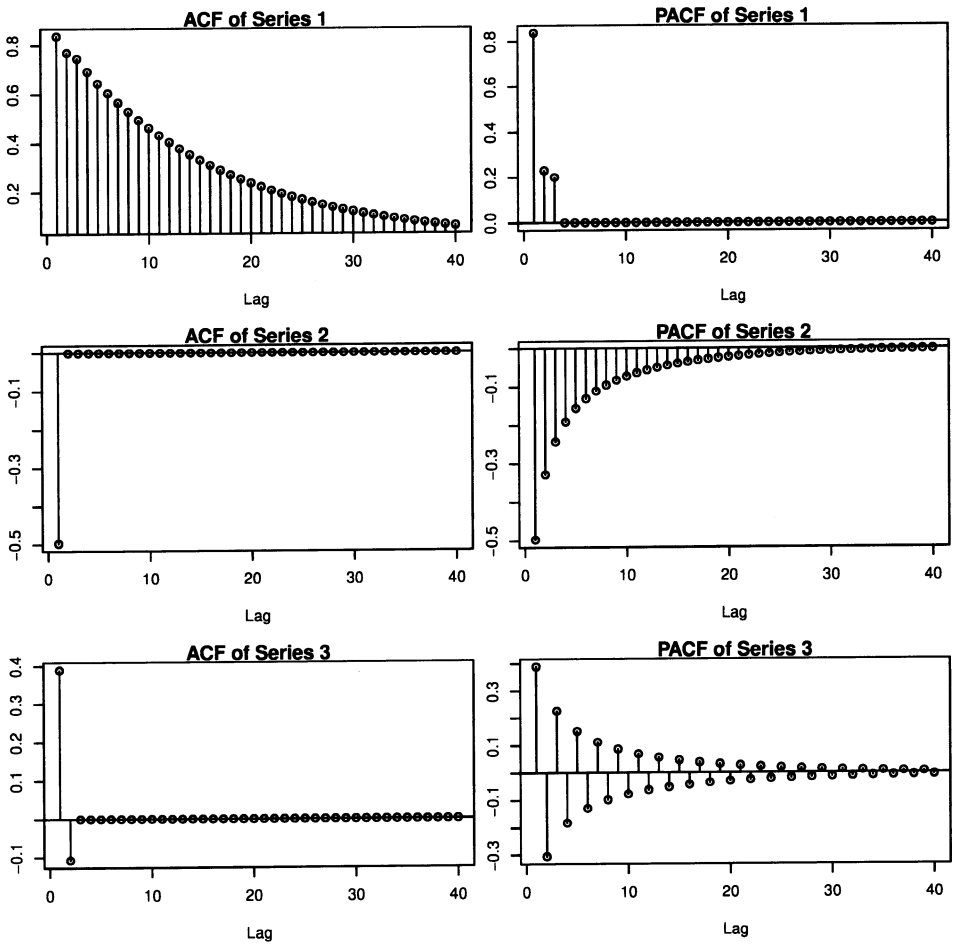
ACF of Series 1
PACF of Series 1
ACF of Series 2
PACF of Series 2
ACF of Series 3
PACF of Series 3
Lag

Figure 1

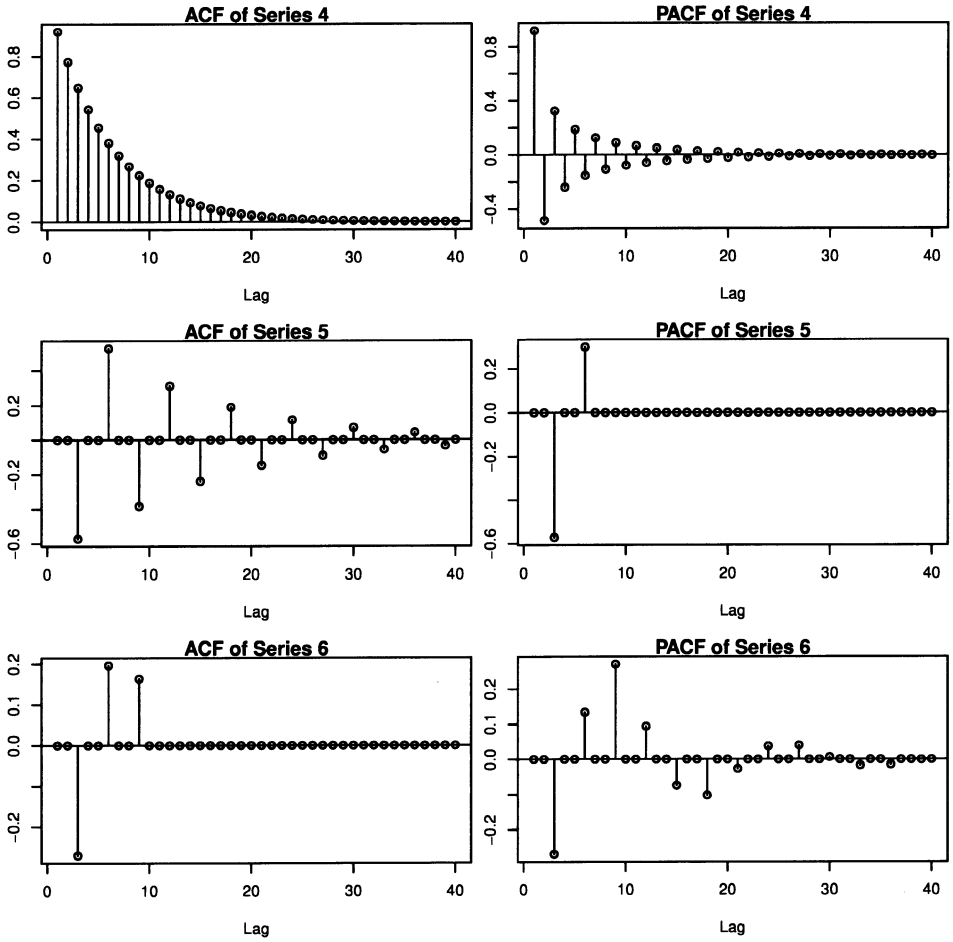
ACF of Series 4
PACF of Series 4
ACF of Series 5
PACF of Series 5
ACF of Series 6
PACF of Series 6
Lag

Figure 2