Aalto University School of Science
Mat-1.1631 Mathematics 3-I
Alestalo

Exam 17.2.2014.
Please fill in all the required information to each exam paper.

No calculators are allowed.

1. The function $f\colon \mathbf{R} \to \mathbf{R}$ is 2π-periodic, and $f(x) = x$ for $-\pi \le x \le \pi$.
   a) Sketch the graph of f on the interval $-3\pi \le x \le 3\pi$.
   b) Calculate the Fourier coefficients of f and write down the first three non-zero terms from the series.

2. Let $u(x, y) = 2x - 3y + 4$ for $(x, y) \in \mathbf{R}^2$. Find a function v so that the function $f(x + iy) = u(x, y) + iv(x, y)$ is analytic in the whole complex plane $\mathbf{C}$, and give a formula for $f(z)$ in terms of the variable $z = x + iy$.

3. Find all complex solutions of the equation $z^3 + 8 = 0$. You may give the answers in the polar form.

4. a) Give a short explanation to the formula

$$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

   for x real.
   b) Calculate the value of the complex number $\ln(3 - 3i)$.

5. Calculate the integrals

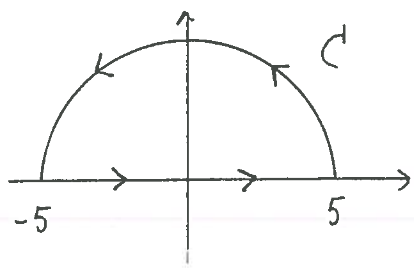
$$\int_C \frac{dz}{z} \quad \text{and} \quad \int_C \frac{dz}{\bar{z}}$$

   where C is the unit circle with the positive orientation.

6. Using the residue integration method, calculate the integral

$$\int_C \frac{e^{iz}}{9 + z^2}\, dz,$$

   where C is the closed semi-circle in the upper half-plane with radius equal to 5 and center at the origin.



Some useful formulas on the backside!

Formulas related to Fourier series

Let $f\colon \mathbf{R} \to \mathbf{R}$ be piecewise continuous and $2L$-periodic. Then

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right),$$

where

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right)\, dx \quad \text{and} \quad b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right)\, dx$$

for $n \geq 1$, and

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx.$$