Aalto University School of Science
Mat-1.1631 Mathematics 3-I
Alestalo

Exam 12.11.2014.

Please fill in all the required information to each exam paper.

No calculators are allowed.

1. The function $f\colon \mathbf{R} \to \mathbf{R}$ is 2π-periodic, and $f(x) = |x|$ for $-\pi \leq x \leq \pi$.
   a) Sketch the graph of f on the interval $-3\pi \leq x \leq 3\pi$.
   b) Calculate the Fourier coefficients of f and write down the first three non-zero terms from the series.

2. Let $u(x,y) = 2x^2 - 2y^2 - 3y$ for $(x,y) \in \mathbf{R}^2$. Find a function v so that the function $f(x+iy) = u(x,y) + iv(x,y)$ is analytic in the whole complex plane $\mathbf{C}$.

3. Find all complex solutions of the equation $z^4 + 81 = 0$. You may give the answers in the polar form.

4. a) Give a short explanation to the formula

$$\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

   for x real.
   b) Calculate the value of the complex number $\ln(-2-2i)$.

5. Calculate the integrals

$$\int_C \frac{dz}{z} \quad \text{and} \quad \int_C \frac{dz}{\overline{z}}$$

   where C is the unit circle with the positive orientation.

6. Using the residue integration method, calculate the integral

$$\int_C \frac{e^{iz}}{16+z^2}\,dz,$$

   where C is the closed semi-circle in the upper half-plane with radius equal to 5 and center at the origin.

Some useful formulas on the backside!

Formulas related to Fourier series

Let $f\colon \mathbf{R} \to \mathbf{R}$ be piecewise continuous and $2L$-periodic. Then

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right),$$

where

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx \quad \text{and} \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$$

for $n \geq 1$, and

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx.$$