Aalto University School of Science
Department of Mathematics and Systems Analysis

Alestalo

Mat-1.1632 Mathematics 3-II

Examination 26.2.2014

Please fill in all the required information to each exam paper.
No calculators nor formula books are allowed.

Choose five (5) problems!

1. Let
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 0 \\ 3 & 2 & 1 \end{bmatrix}.$$
Show that $\lambda = 3$ is an eigenvalue of A and find a corresponding eigenvector.

2. Let
$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 7 \\ -3 & 2 & 3 \end{bmatrix}.$$
a) Does the Gershgorin theorem imply that A is non-singular (that is, all eigenvalues are different from zero)?
b) Find the LU-decomposition $A = LU$.

3. Solve the system $\mathbf{y}' = A\mathbf{y}$ with initial condition $\mathbf{y}(0) = [0, 5]^T$, where
$$A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}.$$

4. Find the critical point (= equilibrium point) of the system
$$\begin{cases} y_1' = y_1 + 3y_2 + 2 \\ y_2' = 2y_1 + 2y_2 - 4 \end{cases}$$
and determine its type and stability.
Note: It is not required to solve the system, although this is one (accepted) way to obtain the answer.

5. Using the Laplace transform, solve the initial value problem $y''+6y'+5y = 0$, $y(0) = 0,\ y'(0) = 8$.

6. Find the inverse Laplace transforms of the following expressions:
$$\text{a)}\ \frac{s}{s^2 + 6s + 10}, \quad \text{b)}\ \frac{e^{-2\pi s}}{s^2 + 6s + 10}.$$

Formulas related to the Laplace transform: Please turn over!

Formulas:

Notation: Given $f(t)$, let $F(s) = (\mathcal{L}f)(s)$. Let $u(t)$ = Heaviside step-function and $\delta(t)$ = Dirac delta-function.

$$(\mathcal{L}f')(s) = sF(s) - f(0), \quad (\mathcal{L}f'')(s) = s^2F(s) - sf(0) - f'(0),$$

$$(\mathcal{L}f^{(n)})(s) = s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0),$$

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\}(s) = \frac{1}{s}F(s), \quad \mathcal{L}(f*g) = (\mathcal{L}f)(\mathcal{L}g),$$

where $(f*g)(t) = \int_0^t f(t-\tau)g(\tau)\,d\tau = (g*f)(t)$;

$$\mathcal{L}\{e^{at}f(t)\}(s) = F(s-a), \quad \mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s).$$

Transforms:

$f(t)$	$F(s)$
$\delta(t-a)$	e^{-as}
$u(t-a)$	e^{-as}/s
1	$1/s$
t^n	$n!/s^{n+1}$
e^{at}	$1/(s-a)$
$\sin \omega t$	$\omega/(s^2+\omega^2)$
$\cos \omega t$	$s/(s^2+\omega^2)$
$\sinh at$	$a/(s^2-a^2)$
$\cosh at$	$s/(s^2-a^2)$