

Helsinki University of Technology
Mat-1.461 Mathematics I

M. Mataich

Third mid-term Exam. 14/December/2004

Please write on sheet on separate lines:

- 1) course title, date
- 2) student number + letter, **IN BLOCK LETTERS** surname underlined, all given names
- 3) signature

1. Calculate the following integrals.

(a) $\int \frac{1}{1-x^2} dx$

(b) $\int \sec^2 x \cosh(\tan x) dx$

(c) $\int \frac{1}{x \ln \sqrt{x}} dx$

2. (a) Approximate $\sin(1) = \int_0^1 \cos x dx$, using (a) Midpoint Rule, (b) Trapezoidal Rule and (c) Simpson's Rule with $n = 4$.

(b) Locate and classify all local extrema of $f(x) = \int_0^x (t^2 - 3t + 2) dt$.

3. (a) Suppose that the birth rate for a certain population is $b(t) = 2e^{0.04t}$ million people per year, and the death rate for the same population is $d(t) = 2e^{0.02t}$ million people per year. Show that $b(t) \geq d(t)$ for $t \geq 0$, and explain why the area between the curves represents the increase in population. Compute the increase in population for $0 \leq t \leq 10$.

(b) A pottery jar has circular cross sections of radius $4 + \sin \frac{x}{2}$ inches for $0 \leq x \leq 2\pi$. Sketch a picture of the jar and compute its volume.

4. Determine whether the integral

$$\int_{-\infty}^0 x e^x dx$$

converges or diverges. Find its value if it converges.