Helsinki University of Technology
Mat-1.462 Mathematics II

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Please write on sheet on separate lines:
1)course title, date
2)student number + letter, IN BLOCK LETTERS surname underlined, all given names
3) signature

1. Use the definition of the limit for multivariable function to prove that

$$\lim_{(x,y,z)\to(0,0,0)} e^{\sqrt{x^2+y^2+z^2}} = 1$$

2. A certain gas satisfies the law $PV = T - \frac{4P}{T^2}$ where P =pressure, V = volume, and T = temperature.

   (a) Calculate $\frac{\partial T}{\partial P}$ and $\frac{\partial T}{\partial V}$ at the point where $P = V = 1$ and $T = 2$.

   (b) If measurements of P and V yield the values $P = 1 \pm 0.0001$ and $V = 1 \pm 0.002$, find the approximate error in the calculated value of T.

4. Solve the initial value problem

$$\begin{cases} y'' + y' - 2y &= 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

5. Show that

$$\int_{(1,2)}^{(3,4)} (6xy^2 - y^3)\,dx + (6x^2y - 3xy^2)\,dy$$

   is independent of the path of integration and evaluate it.

6. Use Green's Theorem to evaluate the integral.

$$\oint_C (2xy - x^2)dx + (x + y^2)dy$$

   assume that the curve C is oriented counterclockwise, where C is the closed curve of the region bounded by $y = x^2$ and $y = \sqrt{x}$.