Department of Mathematics and Systems Analysis
Aalto University, School of Science

Metsalo/Apetrei

MS-A0210 Mathematics 1

2nd midterm exam, 10 December 2013

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Degree Programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

Calculators are not allowed. The examination time is 3 hours.
Ask, if You suspect typos in the text!

1. For each $c \in \mathbf{R}$ the equation $y = 1 + c^2 - 2cx$ gives a straight line in the xy-plane. Give the equation for the envelope of this family of lines on the form $y = g(x)$.

2. Let H be a homogeneous solid with volume $V = \iiint_H dV$. The z-coordinate $\bar{z}$ of its centroid is given by $\bar{z} = \frac{1}{V} \iiint_H z \cdot dV$ (and correspondingly for the x- and y-coordinates of its centroid).
The hemisphere $H = \{(x, y, z) \in \mathbf{R} | x^2 + y^2 + z^2 \leq R^2, z \geq 0\}$ has volume $V = 2\pi R^3/3$. Because of symmetry its centroid is on the z-axis. Calculate the z-coordinate $\bar{z}$ of its centroid using
a) cylindrical coordinates,
b) spherical cvoordinates.

3. Calculate the area of the part of the surface $z = f(x, y) = \sqrt{2xy}$, that has the rectangle $1 \leq x \leq 4, 1 \leq y \leq 9$ as its projection down onto the xy-plane.

4. Show that among all rectangular boxes with a total surface area A of its six rectangular faces, the cube has the largest volume. (Its volume is ofcourse $V = (\sqrt{A/6})^3$.)