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Metsalo

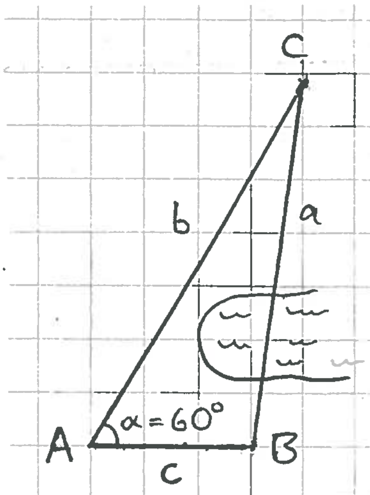
MS-A0210 Mathematics 1

Final exam, 8 January 2014

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Degree Programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

Calculators are not allowed. The examination time is 3 hours.
Ask, if You suspect typos in the text!

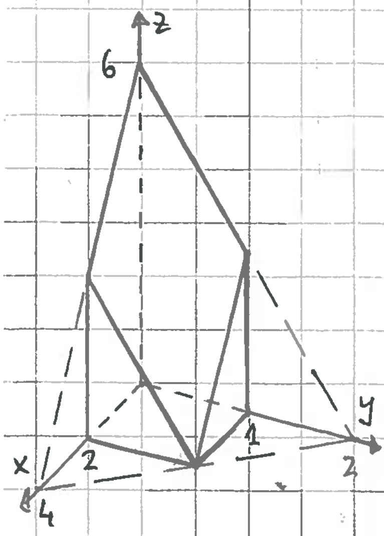
1. We wanted to measure the distance a between two points B and C. For this purpose we used a sextant, which gives the angle 60° with high accuracy and walked around, until we found a point A such that the angle between AB and AC was 60°. We then measured AB to be $c = 300 \pm 5m$ and AC to be $b = 800 \pm 10m$. The law of cosines $a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$ then gave that the distance BC is $a \approx 700m$.
   Use the differential to calculate an approximate upper limit for the uncertainity in this approximation of a due to the uncertainities in b and c.



2. The equation $x+2y+z+e^{2z} = 1$ determines implicitely a function $z = f(x, y)$ in a neightbourhood of the origin $(x, y) = (0, 0)$ such that $f(0, 0) = 0$.
   a) Find the Maclaurin-polynomial (Taylor-polynomial near the origin $(0,0)$) $P_1(x, y)$ of degree 1 for $f(x, y)$.
   b) Find the Maclaurin-polynomial (Taylor-polynomial near the origin $(0,0)$) $P_2(x, y)$ of degree 2 for $f(x, y)$.

3. Use the method of Lagrange multipliers to find the maximum and minimum values of the function $x^2 + xy + \frac{7}{4}y^2$ on the ellipse $x^2 + xy + 2y^2 = 1$.

4. The hexahedron to the right is bounded by the coordinate planes, the planes $x = 2, y = 1$ and the plane, that passes through the points (4,0,0), (0,2,0) and (0,0,6), so its volume is 6. At the point (x, y, z) the density of the hexahedron is $\delta(x, y, z) = xy$, so $\delta_{min} = 0$ and $\delta_{max} = 2$. Calculate the mass of the hexahedron.



5. Calculate the area of the part of the paraboloid $z = 1-(x^2+y^2)$, which is above the xy-plane.