Department of Mathematics and Systems Analysis
Aalto University, School of Science

Metsalo

MS-A0210 Mathematics 1

Final exam, 4 Sept 2014

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Degree Programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

Calculators are not allowed. The examination time is 3 hours.
Ask, if You suspect typos in the text!

1. Let us study the surface $xyz = 2$ in the 1^{st} octant $x, y, z > 0$. If we choose an arbitrary point on the surface, then the tangent plane to the surface at that point bounds a tetrahedron together with the coordinate planes. Show that the volume of the tetrahedron is independent of the chosen point and calculate this volume.

2. $f(x, y, z) = x \cdot e^{y+z^2} \Rightarrow f(2, -4, 2) = 2$. Approximate $f(2.05, -3.92, 1.97)$ using linear approximation.

3. Let $f(x, y)$ be a continous function with continous 1^{st} order partial derivatives in $\mathbf{R}^2$. Introduce polar coordinates r and θ via $x = r\cos\theta, y = r\sin\theta$. Then $f(x, y) = f(r\cos\theta, r\sin\theta) = F(r, \theta)$. Show that $(\partial F/\partial r)^2 + (\frac{1}{r} \cdot \partial F/\partial\theta)^2 = (\partial f/\partial x)^2 + (\partial f/\partial y)^2$.

4. Calculate $\int_0^8 (\int_{\sqrt[3]{y}}^2 \sqrt{16 - x^4} dx) dy$.
(Hint: Change of order of integration.)

5. The sphere $x^2 + y^2 + z^2 = R^2$ and the right circular cone $x^2 + y^2 = z^2$ bound a region W in the shape of an icecream cone in the upper halfspace $z \geq 0$. In the figure to the right we have a cross-section of W through its axis of symmetry (the z-axis).
a) Calculate the volume $V = \iiint_W dV$ of W.
(Check: $V < \frac{1}{6} \cdot \frac{4\pi}{3} R^3$, since six icecream cones can be fitted in the sphere $x^2 + y^2 + z^2 = R^2$ and there will be some space left over.)
b) Because of symmetry, the centroid of W is on the z-axis. Calculate the z-coordinate $\bar{z} = \frac{1}{V} \iiint_W z \cdot dV$ of the centroid.
(Hint: Because of the shape of W, cylindrical or spherical coordinates may be suitable.)