Aalto University
Björn Ivarsson, 050-4067 832

Midterm 2, Wednesday 12.03.2014 16:00 - 19:00
Mathematics 2, MS-A0310.
Lecture notes, books, pocket calculators, smart phones or computers are not allowed during the midterm.

Explain your solutions! If you only give the answer you will not get any points.

(1) Calculate the area of the region enclosed by the curve

$$\gamma(t) = (\cos^3 t, \sin^3 t),\ 0 \le t \le 2\pi.$$

(6p)

(2) Let S be the sphere $x^2+y^2+z^2 = 4$ and $F(x, y, z) = (x^2, xz, 3z)$. Let $\vec{N}$ be the unit normal field pointing away from the origin. Calculate

$$\oiint_S F \cdot \vec{N}\, dS.$$

(6p)

(3) Let $F(x, y, z) = (y, z, x)$. Use Stokes's Theorem to show that

$$\oint_\gamma F(x, y, z) \cdot d\gamma = \sqrt{3}\pi a^2$$

where γ is the suitably oriented intersection of the surfaces $x^2 + y^2 + z^2 = a^2$ and $x + y + z = 0$. (6p)

(4) Let

$$F(x, y) = \left(\frac{xy}{x^2+y^2}, \frac{y^2}{x^2+y^2}\right)$$

when $(x, y) \neq (0, 0)$.

(a) Express the vector field in polar coordinates,

$$F = F_R\hat{R} + F_\theta\hat{\theta}.$$

(3p)

(b) Calculate div F in polar coordinates. (3p)

Good luck!