Aalto University
Björn Ivarsson, 050-4067 832

Exam, Wednesday 12.03.2014 16:00 - 19:00
Mathematics 2, MS-A0310.
Lecture notes, books, pocket calculators, smart phones or computers are not allowed during the midterm.

Explain your solutions! If you only give the answer you will not get any points.

(1) Let $F(x,y) = (2xy, x^2)$ and let γ be the curve $x^2 + 4y^2 = 4$ oriented counterclockwise. Calculate

$$\oint_\gamma F(x,y) \cdot d\gamma.$$

(6p)

(2) Calculate the area of the region enclosed by the curve

$$\gamma(t) = (\cos^3 t, \sin^3 t), \ 0 \leq t \leq 2\pi.$$

(6p)

(3) Let S be the sphere $x^2+y^2+z^2 = 4$ and $F(x,y,z) = (x^2, xz, 3z)$. Let $\vec{N}$ be the unit normal field pointing away from the origin. Calculate

$$\oiint_S F \cdot \vec{N} \, dS.$$

(6p)

(4) Let $F(x,y,z) = (y,z,x)$. Use Stokes's Theorem to show that

$$\oint_\gamma F(x,y,z) \cdot d\gamma = \sqrt{3}\pi a^2$$

where γ is the suitably oriented intersection of the surfaces $x^2 + y^2 + z^2 = a^2$ and $x + y + z = 0$. (6p)

Good luck!