Aalto University

MS-A0211 / Period III 2023

Final Exam 20.02.2023

No calculators or notes of any kind are allowed.

This exam consists of 9 problems (total number of points = **10pts**).

Question 1: (1pt) Find a parametric equation for the tangent line to the curve given by

$$r(t) = \left(t, \frac{1}{t}, \sqrt{t^2+1}\right)$$

when $t = 1$.

Question 2: (1pt) Compute the following limit or show it does not exist:

$$\lim_{(x,y)\to(0,0)} e^{\frac{xy}{x^2+y^2}}$$

Question 3: (1pt) Determine if $f(x, y)$ is continuous at $(1, 0)$ where

$$f(x,y) = \begin{cases} \dfrac{(x-1)^2y^2}{(x-1)^2+y^2} & \text{if } (x,y) \neq (1,0) \\ 0 & \text{if } (x,y) = (1,0) \end{cases}$$

Question 4: (1pt) Find a point on the surface of

$$f(x,y) = e^{x-y}(x^2 - y^2) + 1$$

where the tangent plane is horizontal and the equation of the tangent plane.

Question 5: (1pt) Find and classify all the critical points of the function

$$f(x,y) = (x+y)(x^2+y).$$

Question 6: (1pt) Use Lagrange multipliers to find the closest and the furthest points on the curve $y = x^2$ from point $(0, 1)$.

Question 7: (1pt) Reverse the order of integration for

$$\int_0^1 \int_{x^2}^{x} f(x, y)\, dy\, dx.$$

That is, write it as an integral of the form $\iint \ldots\, dx\, dy$.

Question 8: (1pt) Find the volume of the region that lies above $z = x^2 + y^2$ and below the plane $z = 2$.

Question 9: (2pts) Let E be the solid region that lies inside the cylinder $x^2 + y^2 = 1$ with $y \geq x$ and $0 \leq z \leq 3$. Let

$$d(x, y, z) = x^2 + y^2 + z^2$$

be the density of E. Sketch E and write a triple integral in cylindrical coordinates that gives the mass of E. Evaluate the integral.