CS-E5755 Nonlinear Dynamics and Chaos
Exam 3.9.2020

Calculator is allowed, no other material.

Problem 1. (6 p) Analyse the following system. Sketch the vector fields as r is varied. Determine the critical value where the bifurcation occurs. Sketch the bifurcation diagram and determine which bifurcation is in question.

$$\dot{x} = rx + 4x^3$$

Problem 2. (6 p) Analyse the following system and sketch a plausible phase portrait.

$$\begin{aligned} \dot{x} &= x(2 - x - y) \\ \dot{y} &= x - y \end{aligned}$$

Problem 3.
(a) (3 p) Is the following system reversible? ("Yes" or "no" is not enough. Justification is required.)

$$\begin{aligned} \dot{x} &= y(1 - x^2), \\ \dot{y} &= 1 - y^2. \end{aligned}$$

(b) (3 p) In your own words, explain what it means that the value of the fractional dimension of the Lorenz attractor is between 2 and 3 and in fact close to 2.

Problem 4.
(a) (3 p) Find the value/values of r at which the logistic map has a superstable fixed point

$$x_{n+1} = rx_n(1 - x_n).$$

Here, $0 \leq x_n \leq 1 \ \forall n \in \mathbb{Z}$, and $0 \leq r \leq 4$.

(b) (3 p) Find the fixed points and define their stabilities for the cubic map $x_{n+1} = 3x_n - x_n^3$. Set $x_n = 2$, iterate, and see what happens. Name what you found.