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CS-C2160 Theory of Computation

Spring 2023

Exam Fri 21 April 2023, 9–12 a.m.

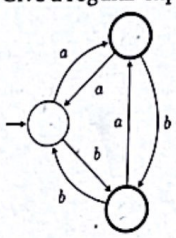
You are allowed to bring with you a single one-sided A4 "cheat sheet", **personally handwritten by you. (NO photocopies, NO printouts, NO computer type-set text.)** Please include your name and student ID at the top of the cheat sheet, and return it together with your answer sheets at the end of the exam.
Note: If you have not completed your computerised home assignments, your exam will not be graded.

1. (a) Design a deterministic finite automaton that recognises the language

$$L = \{w \in \{0,1\}^* \mid \text{the number of 0's in } w \text{ is even (or zero) and } w \text{ ends in a 1}\}.$$

8 points

(b) Give a regular expression that describes the language recognised by the following DFA:



7 points

2. (a) Consider the following context-free grammar G:

$$S \rightarrow ASb \mid \varepsilon$$
$$A \rightarrow aA \mid a$$

Describe the language $L = L(G)$ generated by G mathematically or verbally as simply as you can. Show that G is ambiguous. 5 points

(b) Design an unambiguous grammar equivalent to G, that is an unambiguous G' for which $L(G') = L$. (You do not need to prove the unambiguity of G'.) 5 points

(c) Prove (precisely!) that the language $L = L(G)$ is not regular. 5 points

3. (a) Justify the claim: if languages A and B over the alphabet $\Sigma = \{0,1\}$ are regular, then so is the language $A \cap B$. 7 points

(b) Based on part (a), justify the claim: if a language $L \subseteq \{0,1\}^*$ is regular, then so is the language $L' = \{w \in L \mid w \text{ is of even length (possibly zero)}\}$. 7 points

4. Give a brief but precise justification, based on material presented on the course, for each of the following statements: (i) all regular languages are context-free, (ii) all context-free languages are decidable, (iii) all decidable languages are semidecidable, (iv) there exist languages that are not semidecidable. 16points

Total 60 points