CIV-E4100 Stability of Structures
Examination, April 20, 2023 / Niiranen (Baroudi)

This examination consists of 3 problems rated by the standard scale 1...6. Neither course material nor calculators are allowed – pen(cil) and paper only.

Problem 1

The governing differential equation of the *flexural buckling* problem of elastic beams (columns or beam-columns) following the *Euler–Bernoulli beam* theory can be written in the form

$$(EIw'')''(x) - (N^0 w')'(x) = 0,\ 0 < x < L.$$

(i) Briefly explain, preferably with a few formulae, how to solve the critical buckling load.

(ii) Briefly explain, preferably with a few formulae, the origin of the second term in the left hand side.

(iii) (1) How should one modify the differential equation when modeling a beam lying on an elastic foundation? (2) How could one take into account eccentric compressive end point loadings?

Problem 2

Let us consider the pure bending of a beam having a rectangular cross section (width b, height $h > b$): deflection and lateral rotation are supported at both ends, bending moments are acting at both ends (as depicted in Figure 1).

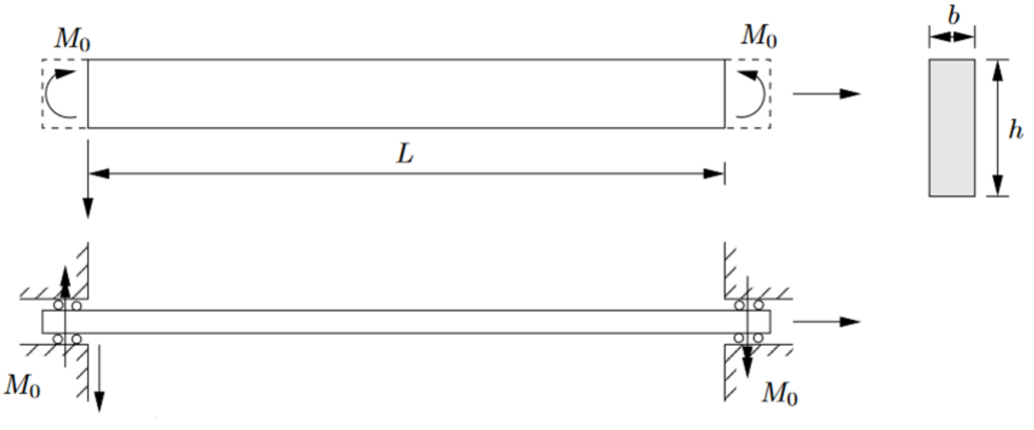

Figure 1: Simply supported beam

The governing differential equations of the *lateral-torsional buckling* problem of elastic beams following the *Euler–Bernoulli beam* theory can be written in the form

$$(EIw'')''(x) - (M^0\phi')'(x) = 0,$$
$$(EI_\omega\phi'')''(x) - (GI_t\phi')'(x) - (M^0w')'(x) = 0,\ 0 < x < L.$$

Correspondingly, the increment of the total potential energy can be written in the form

$$\Delta\Pi = \frac{1}{2}\int_0^L \big(EI(w'')^2 + GI_t(\phi')^2 + EI_\omega(\phi'')^2 + (M^0\phi)'w'\big)dx.$$

(i) Find the exact solution or a reasonable approximation for the critical bending moment by using either (1) the differential equations or (2) the energy expression.

(ii) Briefly explain, preferably with a few formulae, how to take into account the self-weight of the beam in the model.

Problem 3

Let us consider the buckling of a rectangular plate (widths a and b): deflection is supported along two opposite boundary lines, whereas one boundary line is clamped and one boundary line is free; uniformly distributed in-plane, compressive edge loadings (equal to each other) act on the simply supported boundaries.

The increment of the total potential energy following the *Kirchhoff plate* theory can be written in the form

$$\begin{aligned}\Delta\Pi =& \frac{D}{2}\int_\Omega \big(w_{,xx}^2 + w_{,yy}^2 + 2\nu w_{,xx}w_{,yy} + 2(1-\nu)w_{,xy}^2\big)dxdy \\ &+ \frac{1}{2}\int_\Omega \big(N_{xx}^0 w_{,x}^2 + N_{yy}^0 w_{,y}^2 + 2N_{xy}^0 w_{,x}w_{,y}\big)dxdy,\end{aligned}$$

where the bending rigidity D and Poisson's ratio ν are assumed to be constants.

(i) Find the exact solution or a reasonable approximation for the critical edge loading by using the energy expression.

(ii) Briefly explain, preferably with a few formulae, how to take into account a beam stiffener in the model (in the compression direction).