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CS-C2160 Theory of Computation — Spring 2023

Exam Fri 9 June 2023, 9–12 a.m.

You are allowed to bring with you a single one-sided A4 "cheat sheet", **personally handwritten by you.** (**NO photocopies, NO printouts, NO computer type-set text.**) Please include your name and student ID at the top of the cheat sheet, and return it together with your answer sheets at the end of the exam.
Note: If you have not completed your computerised home assignments, your exam will not be graded.

1. Show that the following languages are regular, by describing each one in terms of either a finite automaton or a regular expression:

   (a) $\{w \in \{0,1\}^* \mid w$ contains three consequent zeros or three consequent ones (or both)$\}$;

   (b) $\{w \in \{0,1\}^* \mid w$ contains neither three consequent zeros nor three consequent ones$\}$;

   (c) $\{w \in \{0,1\}^* \mid$ the number of ones in w is a multiple of three (possibly zero)$\}$;

   (d) $\{w \in \{0,1\}^* \mid |w| \geq 3$ and the third-to-last symbol in w is a 1$\}$. 16 points

2. (a) Design a context-free grammar describing balanced sequences of parentheses that may also contain parallel subexpressions, e.g. "(()())()" or "()()()". Based on your grammar, give parse trees for the above sequences.

   (b) Prove (precisely!) that the language discussed in part (a) can not be described by a regular expression. 15 points

3. Justify the claim: if languages A and B over an alphabet Σ are context-free, then so are the following languages:

$$\begin{aligned} A \cup B &= \{w \in \Sigma^* \mid w \in A \text{ or } w \in B\}, \\ AB &= \{w \in \Sigma^* \mid w = xy \text{ for some } x \in A, y \in B\}, \\ A^* &= \bigcup_{k \geq 0} A^k = \{w_1 \ldots w_k \mid k \geq 0,\ w_i \in A \text{ for all } i = 1, \ldots, k\}. \end{aligned}$$

   Conclude from the above result that all regular languages are context-free. (*Hint:* Consider regular expressions.) 15 points

4. Which of the following claims are true and which are false? (No proofs are needed, just indicate your choice by the letter T or F.)

   (a) The complement of any regular language is context-free.

   (b) The intersection of any two context-free languages is regular.

   (c) Deterministic pushdown automata can recognise some nonregular languages.

   (d) Every decidable language can be generated by a context-free grammar.

   (e) Any context-free language can be recognised by a deterministic Turing machine.

   (f) Nondeterministic Turing machines can recognise some undecidable languages.

   (g) The complement of an undecidable language can be semidecidable.

   14 points

Total 60 points