

**PROBLEMS:**

1. Short tasks on three different topics:

(a) Describe the properties of the electromagnetic radiation of an oscillating electric dipole. (2 p)

(b) Explain the index ellipsoid. How to find the normal modes and their refractive indices in a uniaxial crystal for a given direction of the wave vector by using the index ellipsoid? (3 p)

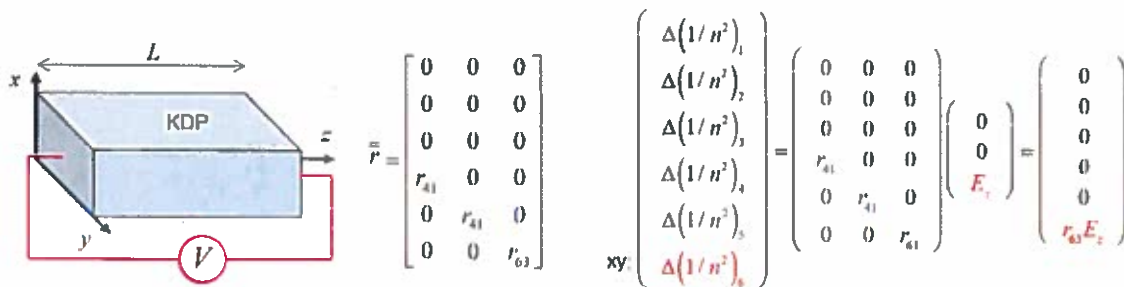
(c) What are the polarization  $\mathbf{P}$  and magnetization  $\mathbf{M}$  of a medium in terms of the electric and magnetic dipole moments excited in the medium? (1 p)

2. Describe qualitatively the Rayleigh-Sommerfeld, Fresnel, and Fraunhofer diffraction (3 p). What does the van Cittert-Zernike theorem state (2 p)? Plot schematically the magnitude of the far-field degree of spatial coherence  $\gamma(r, r_0 = 0)$  of light radiated by a circular incoherent source; here  $r$  is the radial distance from the central point of the intensity profile at  $r_0 = 0$  (1 p).

3. A uniaxial KDP crystal in a Pockels cell shown in the figure below is used as a phase retarder or a polarization modulator for an optical beam propagating along the z-axis.

(a) Assume that the applied voltage is equal to the quarter-wave voltage,  $V_{\pi/2}$ , and find the output polarization of the beam if the incident beam is (1) linearly polarized along the x-axis, (2) linearly polarized at  $45^\circ$  to the x-axis, and (3) right circularly polarized. (3 p)

(b) Describe shortly two different methods to obtain an electro-optic intensity modulator based on this device. (3 p)



4. Artificial colors by Rayleigh scattering at metal nanoparticles embedded in a dielectric host medium: What dielectric constant of the host medium for silver nanoparticles would be needed to obtain the plasmon-resonance wavelength at blue, green, and red wavelengths of 470 nm, 530 nm, and 650 nm, respectively? Repeat the calculations for gold nanoparticles, neglecting the imaginary part of the electric permittivity. At what wavelength the neglectation is not appropriate? The complex refractive indices of silver and gold at the wavelengths of interest are given in the table below. (6 p)

	470 nm	530 nm	650 nm
Ag	$0.05 + 2.85i$	$0.05 + 3.41i$	$0.05 + 4.41i$
Au	$1.31 + 1.85i$	$0.56 + 2.20i$	$0.16 + 3.60i$

5. (a) A LED emits light of Lorentzian spectrum with a linewidth  $\delta\nu = 1013$  Hz centered about wavelength  $\lambda_0 = 700$  nm. What is the maximum time delay within which the magnitude of degree of coherence  $\gamma_{12}$  is greater than 0.5? What is the coherence length of the source? (3 p)

(b) In Young's double-pinhole experiment, the light source is a small circular hole of diameter 0.1 mm in front of a fully spatially incoherent sodium lamp ( $\lambda_0 = 589.3$  nm). If the distance from the source to the double-pinhole screen is 10 m, how far apart will the pinholes be when the fringe pattern disappears? (3 p)

**Formulas that may be useful:**

$$\lambda = \frac{c}{\nu}$$

$$\sum_{k=0}^n ar^k = a \left( \frac{1-r^{n+1}}{1-r} \right)$$

$$f = \frac{\rho_m^2}{m\lambda} = \frac{\rho_1^2}{\lambda}$$

$$E_m \approx E_1 \approx 2E_{\text{incident}}$$

$$x_{\min} = f\Delta\theta_{\min} = 1.22 \frac{\lambda f}{D}$$

$$\alpha = \frac{p}{\epsilon_0 E} = \frac{-e^2}{2m\omega_0\epsilon_0} \frac{\delta^2 - i\frac{\gamma}{2}}{\delta^2 + \left(\frac{\gamma}{2}\right)^2}$$

$$\text{Re}[z] = \frac{z+z^*}{2}$$

$$\text{Im}[z] = \frac{z-z^*}{2i}$$

$$k_{x,SP}^2 = k^2 \frac{\epsilon_1\epsilon_2}{\epsilon_1 + \epsilon_2}$$

$$FT\{\exp(-t^2/\tau^2)\} = \sqrt{\pi/\tau^2} \exp(-\pi^2\nu^2/\tau^2)$$

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2} \text{Re}[\gamma_{12}]$$

$$p = 4\pi\epsilon\alpha^3 \frac{\epsilon_{np}(\omega) - \epsilon}{\epsilon_{np}(\omega) + 2\epsilon}$$

$$\mathcal{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} |\gamma_{12}|$$

$$J_1(1.22\pi) = 0$$

$$\gamma(r) = \gamma(x, y) = \frac{\iint_{x'^2+y'^2 < a^2} I_0 e^{i\frac{k}{R}(xx'+yy')} dx' dy'}{\iint_{x'^2+y'^2 < a^2} I_0 dx' dy'} = 2 \frac{J_1(kra/R)}{kra/R}$$

$$FT\left\{\frac{1}{\pi(\nu - \nu_0)^2 + (\delta\nu/2)^2}\right\} = \exp(-i2\pi\nu_0 t - \pi\delta\nu|t|)$$

$$U_{P_0} = \frac{i}{\lambda} \oint_{\Sigma} U(P_1) \frac{e^{ikr}}{r} \left[ \frac{\cos(\vec{n}, \vec{r}) - \cos(\vec{n}, \vec{r}')}{2} \right] dS$$

$$U(x_0, y_0) = \frac{i}{\lambda} \frac{e^{ikz}}{z} \iint_{-\infty}^{\infty} U(x_1, y_1) e^{\frac{ik}{2z}[(x_0-x_1)^2 + (y_0-y_1)^2]} dx_1 dy_1$$

$$U(x_0, y_0) = A \iint_{-\infty}^{\infty} U(x_1, y_1) e^{\frac{-ik}{z}[x_0x_1 + y_0y_1]} dx_1 dy_1$$

$$V_{\pi} = \frac{\lambda_0}{2n_0^3 r_{63}}$$

$$\Delta\phi = \pi V/V_{\pi}$$