PHYS-C0256 - Thermodynamics and Statistical Physics
Exam on February 22, 2023

7 problems - 30 points

1. (Concepts, 3 points) Write simple explanation for each of the following
(a) Thermodynamic limit
(b) Clausius inequality
(c) Third law of thermodynamics

2. (Probabilities, 3 points) There are 10 freely moving non-interacting particles in a box with volume V, see Fig. 1. A wall is inserted such that it divides the volume of the box into two equal halves (volumes V/2), say "left" and "right" half. Find the probability that after insertion of the wall
(a) there are no particles in the "left" half.
(b) only one particle is found in the "left" half.
(c) 5 particles are found on each side.

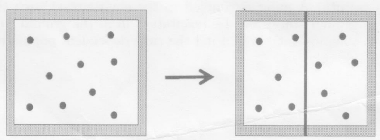

FIG. 1. Problem 2.

3. (Canonical distribution, partition function, 4 points) A three level system (see Fig. 2) has energy level separation $\Delta E = E_e - E_g$ between the first excited state (e) and the ground state (g). There is the third level (f) for which $E_f - E_e = 0.9\Delta E$. The system is canonical as it is coupled to a heat bath at temperature T. Find the equilibrium occupation probabilities p_f, p_e, p_g in each state. Estimate how low should the temperature $(k_B T/\Delta E)$ be to have $p_e < 10^{-3}$. $[\ln(10) = 2.3\ldots]$

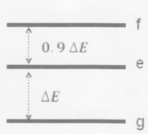

FIG. 2. Problem 3.

4. (Ideal gas, 4 points) Find the ratio of the isothermal and adiabatic compressibilities κ_T/κ_S.

5. (Heat, adiabatic process, entropy, 6 points) Consider one mole of van der Waals gas with equation of state $p = \frac{RT}{V-b} - \frac{a}{V^2}$, where a, b and R are constants.
(a) Calculate the amount of heat produced in a reversible isothermal expansion from V_1 to V_2.
(b) Show that in an adiabatic process $T(V-b)^{(R/C_V)} = const.$, where c_v is heat capacity for one mole of gas at constant volume.
(c) Calculate the entropy of this gas.

6. (a) (Normal metal-Insulator-Superconductor NIS junction, 5 points) For this junction at low temperature T, $eV, k_BT \ll \Delta$, starting from the equation $I = \frac{1}{eR_T}\int_{-\infty}^{\infty} d\epsilon n_S(\epsilon)[f_N(\epsilon - eV) - f_S(\epsilon)]$, show that the following equation holds approximately for its current I versus voltage V dependence,

$$I = I_0 e^{-(\Delta - eV)/k_BT}, \tag{1}$$

where $I_0 = \frac{1}{eR_T}\sqrt{\frac{\pi \Delta k_B T}{2}}$, Δ the superconducting gap, and $n_S(\epsilon)$ is the density of states of the superconductor. Show further that $\frac{d}{dV}(\ln I)$ gives temperature dictated just by constants of nature and temperature.

(b) (Efficiency of a refrigerator, 2 points) The cooling power of N in the same configuration is maximized at $V \simeq \Delta/e$. A more precise calculation shows that the current at this point is $I = 0.48\frac{\Delta}{eR_T}\sqrt{k_BT/\Delta}$ and the cooling power has value $\dot{Q}_{\mathrm{NIS}} = 0.59\frac{\Delta^2}{e^2R_T}(k_BT/\Delta)^{3/2}$. Write the efficiency η of the NIS refrigerator at this optimum point.

7. (Qubit relaxation, 3 points) A qubit is coupled to the environment such that it has transition rates $\Gamma_\downarrow$ (relaxation from the excited to ground state) and $\Gamma_\uparrow$ (excitation from the ground to excited state). The qubit is initiated in the excited state at time instant $t = 0$. Find the time dependent population of the qubit in the ground state after the initialization.