PHYS-C0256 - Thermodynamics and Statistical Physics
Exam on April 17, 2023

7 problems - 30 points

1. (Concepts, 3 points) Write a simple explanation for each of the following:
(a) Micro-canonical ensemble
(b) Clausius inequality
(c) Maxwell demon

2. (Probabilities, 4 points) There are two freely moving non-interacting particles in a box with volume V, see Fig. 1. A wall is inserted such that it divides the volume of the box into two parts, volumes $V_{\text{left}} = V/10, V_{\text{right}} = 9V/10$. Find the probability that after insertion of the wall there are (a) 0, (b) 1, (c) 2 particles in the "left" subdivision.

3. (Canonical distribution, partition function, 6 points) A quantum harmonic oscillator has equidistant energy levels (separation between the levels $\hbar\omega_0$). It is connected to a heat bath at temperature T. Find the expression for the partition function Z. From that then evaluate the internal energy, heat capacity, Helmholtz free energy and entropy. Find the the equilibrium probability p_n to occupy the nth energy level.

4. (Ideal gas, 5 points) Show that the efficiency of the standard Otto cycle (Fig. 1) is $1 - r^{1-\gamma}$, where $r = V_1/V_2$ is the compression ratio and γ is the adiabatic index (ratio of heat capacities at constant pressure and constant volume, respectively).

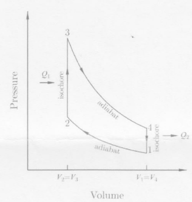

FIG. 1. Otto cycle, Problem 4.

5. (Tunneling, 4 points) Show that the current-voltage dependence of a tunnel junction between two normal conductors (constant density of states) is ohmic, i.e. $I = V/R_{\text{T}}$, where constant R_{T} is the tunnel resistance.

6. (Thermal conduction, 4 points) A uniform rod with cross-sectional area A and length L has temperature (T) dependent thermal conductivity $\kappa = \alpha T$, where α is a constant. It is connected to reservoirs at temperatures T_1 and T_2 at its ends. Find the heat current flowing through the rod.

7. (Qubit relaxation, 4 points) A qubit is coupled to the environment such that it has transition rates $\Gamma_\downarrow$ (relaxation from the excited $|e\rangle$ to ground $|g\rangle$ state) and $\Gamma_\uparrow$ (excitation from the ground to excited state). The qubit is initiated in the state $\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$ at time instant $t = 0$. Find the probability of finding the qubit in the ground state at time t after the initialization.