ELEC-E8740 - Basics of Sensor Fusion — Exam 17.10.2022

Allowed aids:	One (1) A4 sheet with hand-written notes (not photocopied, not written on computer).
Not allowed:	Anything else including lecture notes, slides, textbook, computer, phone, tablet, etc.
Exam paper:	This exam paper needs to be returned along with your answer sheets.

Question 1

Describe briefly (total of about one page) the following terms and what is their significance in sensor fusion:

a) (1p) regularized least squares method

b) (1p) measurement model

c) (1p) cost function

d) (1p) estimation algorithm

e) (1p) Levenberg–Marquardt method

f) (1p) Gaussian distribution

Question 2

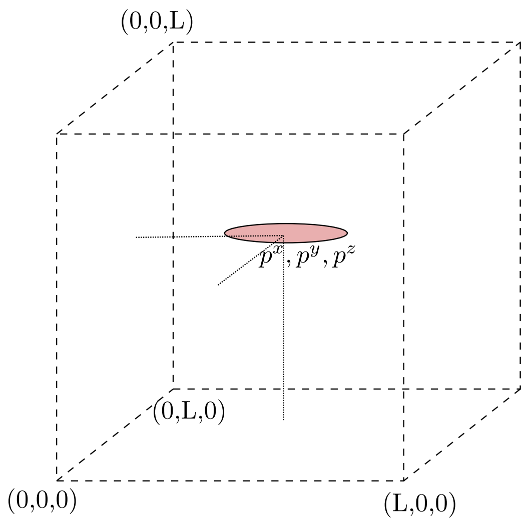

Figure 1: Drone positioning problem for Question 2.

Suppose that we wish to determine a drone position in a closed three dimensional box (p^x, p^y, p^z) as shown in Figure 1. In this scenario, suppose that the drone is equipped with sensors that measure the distance between the drone and the surrounding walls. The measurements from these sensors are given by

$$\begin{aligned} y_1 &= p^x + r_1, \\ y_2 &= p^y + r_2, \\ y_3 &= p^z + r_3, \\ y_4 &= L - p^x + r_4, \\ y_5 &= L - p^y + r_5, \\ y_6 &= L - p^z + r_6, \end{aligned} \tag{1}$$

where r_i for $i = 1, \ldots, 6$ are independent zero-mean random noises with variance σ^2.

a) (2p) Rewrite the measurement model above in a vector notation with the following form:

$$\mathbf{y} = \mathbf{G}\,\mathbf{x} + \mathbf{b} + \mathbf{r}.$$

b) (2p) Which are the minimal subsets of measurements that can be used to find the position of the drone?

c) (2p) If you have all the measurements, what would be a sensible strategy to compute the position in order to minimize the effect of noise?

Question 3

Let us assume that a car as shown in Figure 2 wishes to find its 2D location (x, y) based on distance measurements to four landmarks as illustrated below.

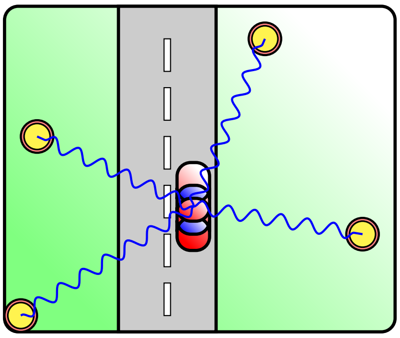

Figure 2: Illustration of car for Question 3.

a) (3p) Assume that the known landmark positions are $(x_i, y_i), i = 1, \ldots, 4$. Formulate the positioning problem as a model of the form

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) + \mathbf{r}, \tag{2}$$

where $\mathbf{y}$ is a vector of measurements, $\mathbf{g}(\mathbf{x})$ is a vector-valued function, $\mathbf{x}$ is the vector of unknowns, and $\mathbf{r}$ contains the measurement noises with joint covariance matrix $\mathbf{R}$.

b) (3p) Identify a suitable cost function for estimating the car location from noisy measurements. Name and briefly explain the underlying idea of three optimization algorithms that could be used to solve the resulting optimization problem.

Question 4

Consider a vector of unknown (static) parameters $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^\mathsf{T}$ and the measurement model function

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} \sin(x_1 + x_2) + x_2 \\ 1 - x_1 \end{bmatrix}.$$

The nonlinear weighted least squares cost function is given by

$$J(\mathbf{x}) = (\mathbf{y} - \mathbf{g}(\mathbf{x}))^\mathsf{T} \mathbf{R}^{-1} (\mathbf{y} - \mathbf{g}(\mathbf{x})).$$

In the Figures 3, the paths taken by two optimization algorithms are shown by red and blue lines.

a) (2p) Explain the principles of gradient descent method and the Gauss-Newton method.

b) (2p) What are the most likely optimization algorithms used in each of Figures 3? Also justify your answer.

c) (2p) Many optimization algorithms require the Jacobian matrix to calculate their descent direction. Derive the Jacobian matrix for the measurement model above.

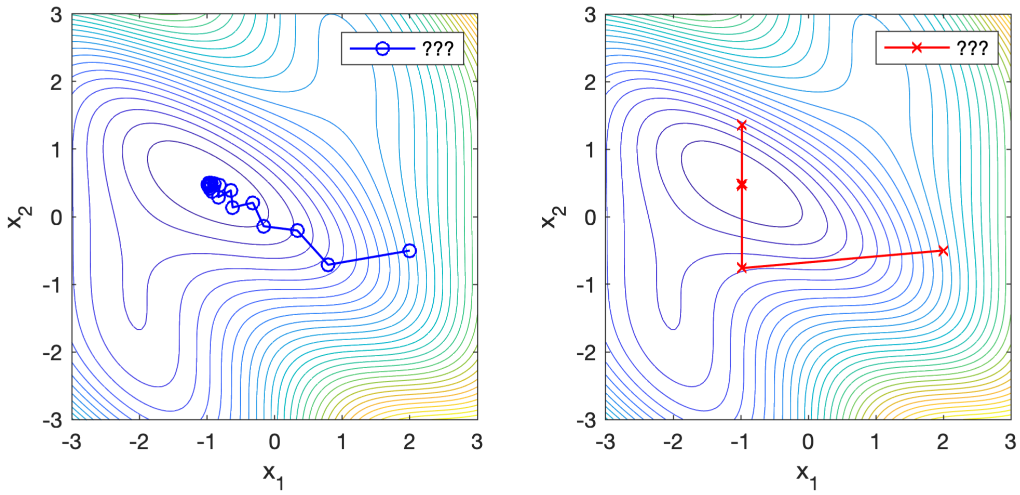

Figure 3: Optimization algorithm results for Question 4.

Question 5

Consider the cost function

$$J(x) = (1.1 - x - \sin(x))^2,$$

where x is a scalar.

a) (3p) Write down a model of the form

$$y = g(x) + r,$$

that is, define y, $g(x)$, and r such that the cost function $J(x)$ would be the nonlinear least squares cost function for this model.

b) (3p) Write down the pseudo-code for minimizing the cost function J(x) by using the gradient descent method.