MEC-E1020 Fluid dynamics - Exam - 19.10.2023

Remember to write your name on your "cheat sheets" and to hand them in with the exam papers. The notes have to be handwritten. If they are not, they should be handed in before you start the exam.

General notes: The answers should be to the point, i.e. they should focus on the question asked. The answers do not have to be long, but they should cover the relevant points from the point of view of the question to get full points. A couple of sentences is typically enough for one point questions, whereas slightly longer answers are expected for two point questions. Lengthy answers with largely irrelevant information that miss the point or that are "beating around the bush" have negative impact on the assessment, as the assessment is also measuring your ability to distinguish between relevant and irrelevant information.

1 Fundamental equations and their solutions

a) An equation is given as

$$\frac{\partial u_j}{\partial x_j} = 0\,.$$

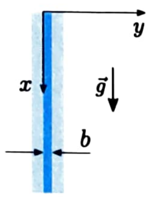
Write this equation in the full component form. What is this equation and what is the physical meaning of the equation and the terms in the equation? (1p)

b) Consider a steady, incompressible, two-dimensional and fully developed flow down a vertical gap between two parallel plates shown in the figure. The gravity is pointing in the direction of the flow.

1. By starting with the continuity and momentum equations in the differential form simplify these equations as much as you can. When dropping out terms, justify this. (1p)
2. Discuss the balance of forces acting on a differential fluid element in this case. (1p)
3. Show that the velocity profile in this case is

$$u(y) = \frac{dp/dx - \rho g}{2\mu} \cdot y(y-b)$$

by solving the momentum equation. (1p)



c) We've studied the conservation equations for three different forms of energy: mechanical energy, thermal/internal energy and total energy. What is the physical meaning and origin of each of the three energy equations and how are these equations linked? (2p)

2 Boundary layers and related flows

a) In the boundary layer equations the wall-normal momentum equation reduces to a condition that pressure does not vary in the direction normal to the wall. Explain, why this is not an exact relation, but only approximately true. Neglect the influence of gravity. (1p)

b) Consider a uniform flow into a straight duct with a constant cross-section. When the flow enters the duct and starts to develop, the velocity outside of the boundary layer does not correspond to the velocity of the uniform flow, but increases with increasing distance from the inlet. Explain the reason for this phenomenon. Would this increase of the velocity depend on the viscosity of the fluid and why? (2p)

c) Give an example of a case without sharp edges or corners, in which flow separation could happen. Explain, why flow separation could happen in this case. (1p)

d) In meandering of rivers the bend of the river increases, as the outer bend is eroded and the resulting sediments are transported along the river bed towards the inner bend. Explain the transport of sediment on the river bed towards the inner bend based on the boundary layer theory. Note that the pressure in the bend increases radially towards the outer bend. (1p)

e) In fully developed pipe flows the shear stress on the wall of the pipe remains constant along the length of the pipe. Is the same true for the shear stress on the wall for flat plate boundary layers and why? (1p)

3 Instability and turbulence

a) Turbulent flow is full of eddies of different size. Discuss the interaction of the eddies with the mean flow and the interaction between eddies of different sizes. (2p)

b) Describe, how the turbulent mixing of momentum is taken into account in the RANS equations, when eddy viscosity based models are used. (1p)

c) Turbulence increases the shear stress on the wall compared to a corresponding laminar flow. Why? (1p)

d) Let's assume that for a flat plate boundary layer

$$c_f \approx \frac{0.027}{\mathrm{Re}_x^{1/7}},$$

where $c_f = 2\tau_0/(\rho U_\infty^2)$ is the nondimensional shear stress on the wall and Re_x is the local Reynolds number based on the distance from the leading edge of the plate x. Consider the boundary layer velocity profile at $x = 10$ m, when the freestream velocity U_∞ is 10 m/s, the density of the fluid is 1000 kg/m^3 and the kinematic viscosity is 1.0×10^{-6} m^2/s. Evaluate the thickness of the viscous sublayer in micrometers at this location. (2p)

4 Numerical techniques

a) The convection-diffusion equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = D\frac{\partial^2 T}{\partial x^2},$$

is approximated with an implicit scheme. Here u and D are known and T is unknown and the spatial derivatives are discretised on time level n. Write the difference approximation for the temporal derivative that produces an implicit scheme with first order accuracy in time. Derive the truncation error for the temporal derivative with this approximation. What is the benefit of using an implicit scheme? (2p)

b) A particular discretisation produces the following modified equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{u^2\Delta t}{2}\frac{\partial^2 T}{\partial x^2} - \frac{u\Delta x^2}{6}\frac{\partial^3 T}{\partial x^3} + \dots .$$

What is the order of accuracy of this method and why? How would you expect this scheme to behave (stable, unstable, mainly diffusive, mainly dispersive) and why? (2p)

c) You need to estimate the numerical uncertainty of your simulation result. Focus on the uncertainty related to the truncation error. How would you proceed? How can you try to check the reliability of your uncertainty estimate? (2p)