**PHYS-E0461 Introduction to plasma physics for fusion and space applications**
**First examination 20.10.2023**

1. **(Plasma phenomena and general aspects)** (6p)
   Give short (not more than 1/3 page for each) but complete answers to following questions:

   (a) Explain what is meant by quasineutrality (in relation to plasmas)? (1p)
   (b) Explain what is the plasma parameter and how it relates to the Debye sphere. (1p)
   (c) Explain what is the magnetic moment. (1p)
   (d) Explain what is meant by the distribution function?(1p)
   (e) What is the difference between 2-fluid and 1-fluid plasmas? (1p)
   (f) During the lectures several plasma waves were introduced. Why there are so many waves compared to neutral gas (that only carries the sound wave)? Give an example of a plasma wave. (1p)

2. **(Magnetic bottle)** (6p)
   Consider a magnetic bottle of length $2a$ with an axial field: $B(z) = B_0[1 + (z/a)^2]$.

   (a) Sketch the field along the $z$-axis. (1p)
   (b) Using conservation of energy and magnetic moment $\mu$, show that a particle (mass $m$) that is mirroring between points $-z_m$ and $z_m$ has parallel velocity given by (3p)

$$v_\parallel = \sqrt{\frac{2\mu B_0}{m}}\sqrt{\left(\frac{z_m}{a}\right)^2 - \left(\frac{z}{a}\right)^2}.$$

   (c) What are the particle velocities $v_\parallel$ and $v_\perp$ at the center of the bottle (where $B = B_0$) and at the mirror points? (2p)

3. **(From kinetic to fluid approach)** (6p)

   (a) Calculate the first 2 velocity moments of the distribution function ■ particle density $n_s$ and plasma flow $n_s\mathbf{V}_s$ – when the velocity space dependence is given by the Maxwellian distribution (normalized to particle density).
   (b) Derive the continuity equation

$$\frac{\partial n_s}{\partial t} + \nabla\cdot(n_s\boldsymbol{V}_s) = \frac{\mathrm{d}n_s}{\mathrm{d}t} + n_s\nabla\cdot\boldsymbol{V}_s = 0$$

   starting from the Vlasov equation

$$\frac{\partial f_s}{\partial t} + \dot{\boldsymbol{r}}\cdot\nabla f_s + \dot{\boldsymbol{v}}\cdot\nabla_v f_s = 0.$$

   Hints: Take the first velocity moment of the Vlasov equation. For the first term you can interchange the derivation and integration. For the second term use equation

$$\nabla\cdot(f\boldsymbol{A}) = f_s\nabla\cdot\boldsymbol{A} + \boldsymbol{A}\cdot\nabla f_s$$

   For the third term use the previous equation and the divergence theorem

$$\int_V \nabla\cdot\mathbf{A}\mathrm{d}V = \int_S \mathrm{d}\boldsymbol{A}\cdot\boldsymbol{S}.$$

   You will also need the convective derivative

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{V}_s\cdot\nabla$$

4. **(MHD wave equation)** (6p)

Take a homogeneous ($p_0$ = constant), stationary ($\boldsymbol{V}_0 = 0$) plasma in a homogeneous magnetic field ($\boldsymbol{B}_0$ = constant). Your task is to linearize and re-organize (use plane waves and Fourier transforms as in the lectures) the set of following MHD equations

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{V} \times \boldsymbol{B})$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}) = 0$$

$$\rho \frac{\partial \boldsymbol{V}}{\partial t} + \rho (\boldsymbol{V} \cdot \nabla) \boldsymbol{V} = -\nabla p + \frac{(\nabla \times \boldsymbol{B})}{\mu_0} \times \boldsymbol{B}$$

$$\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = 0$$

to arrive at a single equation for the perturbed plasma fluid velocity:

$$\left[\omega^2 - \frac{(\boldsymbol{k} \cdot \boldsymbol{B}_0)^2}{\mu_0 \rho_0}\right] \boldsymbol{V}_1 = \left\{\left[\frac{\gamma p_0}{\rho_0} + \frac{B_0^2}{\mu_0 \rho_0}\right] \boldsymbol{k} - \frac{(\boldsymbol{k} \cdot \boldsymbol{B}_0)}{\mu_0 \rho_0} \boldsymbol{B}_0\right\} (\boldsymbol{k} \cdot \boldsymbol{V}_1) - \frac{(\boldsymbol{k} \cdot \boldsymbol{B}_0)(\boldsymbol{V}_1 \cdot \boldsymbol{B}_0)}{\mu_0 \rho_0} \boldsymbol{k}.$$