Advanced Investments
Exam 1
Professor Suominen

October-2023

If the exercise requires mathematics, show all the steps clearly and underscore the final answer. Even if you cannot complete a question, show all your work as partial credit can be given even for incomplete answers. If you feel that some information is missing, make an assumption, state it clearly, and proceed. Last page has some useful formulas that you may want to use.

*** GOOD LUCK ***

Question 1 - 30pts

(a) If S follows the process $dS = \mu S dt + \sigma S dz$, what is the process followed by

(i) $y = S^{-1}$

(ii) $y = \exp(S)$

Express the coefficients of dt and dz in terms of y rather than S (if possible).

Question 2: 20pts

Black-Scholes partial differential equation for a derivative f is

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

When stock price follows a Geometric Brownian Motion, that is the process of S is

$$dS = \mu S dt + \sigma S dz$$

then Black-Scholes call option formula states that the price of the call C is given as

$$C(S, K, r, T-t, \sigma) = S \cdot N(d_1) - K \cdot e^{-r(T-t)} \cdot N(d_2) \qquad [0.1]$$

where $N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}}\, dy$ and $d_1 = \frac{1}{\sigma\sqrt{T-t}} \left(\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right)$ and $d_2 = d_1 - \sigma\sqrt{T-t}$.
The "Greeks" which are partial derivatives of the price with respect to the parameter values

are given as

$$\Delta = \frac{\partial C}{\partial S} = N(d_1)$$

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$$

$$\Theta = \frac{\partial C}{\partial t} = -\frac{S\sigma N'(d_1)}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)$$

$$V = \frac{\partial C}{\partial \sigma} = S\sqrt{T-t}N'(d_1)$$

$$\rho = \frac{\partial C}{\partial r} = K(T-t)e^{-r(T-t)}N(d_2)$$

Use the information given above to show that price of the call option satisfies Black and Scholes partial differential equation.

Question 3: 24pts

Answer TRUE or FALSE (Correct answer +4 pts, Incorrect -4 pts, No answer 0 pts).

(a) Large selling pressure moves prices of liquid stocks more than illiquid stocks

(b) In Kyle (1985) noise traders make the market more liquid.

(c) A stochastic process that follows Geometric Brownian motion $dS = \mu Sdt + \sigma Sdz$ with $\mu > 0$ and $\sigma > 0$, is a martingale.

(d) Etula, Rinne, Suominen and Vaittinen (2020) show that at the turn of the month institutional investors demand for liquidity leads to return reversals

(e) Limits to arbitrage prevent stock market bubbles from appearing

(f) Cox-Ingersoll-Ross interest rate model allows for negative interest rates

Question 4: 26pts

Assume a student is taking this exam and he has answered all other questions (including this one) and knows for a fact he will get full 96 points from all of them but is now pondering whether to answer Question 3f). Assume student's answer to this question is as good as a guess. Assume also that the student has CARA (exponential) utility function with a constant average risk aversion of 0.01, that is $u(x) = -\exp(-0.01x)$. The only input x in the utility function is the points he receives from the this exam.

(a) What is the expected value of student's exam points?

(b) What is his expected utility if he does not answer the last question? Give the answer rounded to a 4 decimals. (e.g. if result 0.54539 then report 0.5454)

(c) What is his expected utility if he answers to the last question? Give the answer rounded to a 4 decimals. (e.g. if result 0.54539 then report 0.5454)

(d) Should the student answer to the Question 3f) (Yes/No)?

Useful formulas:

Itô's lemma:

$$df = \left(\frac{\partial f}{\partial x}\mu(x,t) + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\sigma(x,t)\right)dt + \frac{\partial f}{\partial x}\sigma(x,t)dz$$