1. (a) *Stokes' theorem* states that the surface integral of the curl of a vector field over an open surface is equal to the closed line integral of the vector field along the contour bounding the surface.

   (b) Using *vector phasors* to represent time-harmonic electromagnetic fields, we replace tedious trigonometric expressions, including time derivatives and integrals with respect to time, with complex algebra.

   (c) The *Poynting vector* or power density vector is the instantaneous power flow (magnitude and direction) per unit area. Its SI unit is watts per square meter.

   (d) The *Brewster angle* is the angle of incidence that gives zero reflection. This angle depend on the polarization. For an interface of two non-magnetic media, the Brewster angle always exists for parallel polarization but it does not exist for perpendicular polarization.

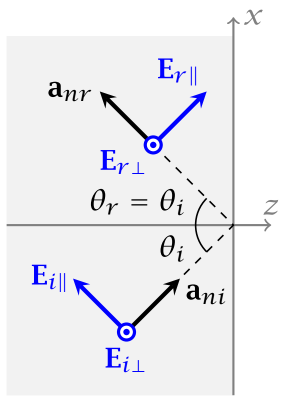
2. (a) Due to spherical symmetry, the electric field must be radial and it can depend on the distance from the charge, so $\mathbf{E} = E_R(R)\,\mathbf{a}_R$ in spherical coordinates. Integrating both sides of Gauss' law over a spherical volume with radius R centered at the origin and applying the divergence theorem gives

$$\int_V \nabla\cdot\mathbf{D}\,dv = \oint_S \mathbf{D}\cdot d\mathbf{s} = \varepsilon_0 E_R \oint_S ds = \varepsilon_0 E_R\, 4\pi R^2$$
$$= \int_V \rho\, dv = q \qquad \Rightarrow \quad E_R = \frac{q}{4\pi\varepsilon_0 R^2} \qquad \Rightarrow \quad \mathbf{E} = \frac{q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R.$$

   (b) Due to rotational symmetry, the magnetic field can only depend on the distance from the current and the magnetic flux lines should form closed loops around the current, so the magnetic field must be on the form $\mathbf{H} = H_\phi(r)\,\mathbf{a}_\phi$. Integrating both sides of Ampère's law over the surface of a disk of radius r centered at the origin in the xy plane and applying Stokes' theorem gives

$$\int_S (\nabla\times\mathbf{H})\cdot d\mathbf{s} = \oint_C \mathbf{H}\cdot d\boldsymbol{\ell} = H_\phi \oint_C d\phi = H_\phi 2\pi r$$
$$= \int_S \Big(\mathbf{J} + \underbrace{\frac{\partial\mathbf{D}}{\partial t}}_{=0}\Big)\cdot d\mathbf{s} = I \qquad \Rightarrow \quad H_\phi = \frac{I}{2\pi r} \qquad \Rightarrow \quad \mathbf{H} = \frac{I}{2\pi r}\mathbf{a}_\phi.$$

3. (a) The phase factor $e^{-j\beta_1(x+z)/\sqrt{2}}$ gives $\mathbf{a}_{ni} = (\mathbf{a}_x + \mathbf{a}_z)$ and the incidence angle $\theta_i = 45°$. The incident wave can be written in the form



$$\mathbf{E}_i = \left(\mathbf{E}_{\mathrm{re}} + j\mathbf{E}_{\mathrm{im}}\right)e^{-j\beta_1(x+z)/\sqrt{2}}, \qquad \begin{cases} \mathbf{E}_{\mathrm{re}} = E_0\mathbf{a}_y \\ \mathbf{E}_{\mathrm{im}} = E_0\dfrac{\mathbf{a}_x - \mathbf{a}_z}{\sqrt{2}} \end{cases}$$

   where the $\mathbf{E}_{\mathrm{re}}$ and E_{im} are orthogonal and equal in magnitude. Rotating the imaginary part ($\mathbf{E}_{i\parallel}$) the shorter direction towards the real part ($\mathbf{E}_{i\perp}$) gives that we have *right-hand circular polarization.*

   (b) Since medium 2 is PEC, we have $\eta_2 = 0$ and $\Gamma = \Gamma_\perp = \Gamma_\parallel = -1$. Deducing the directions from the figure we can write the reflected field

$$\mathbf{E}_r = \left[\mathbf{a}_y + j\frac{\mathbf{a}_x + \mathbf{a}_z}{\sqrt{2}}\right]\Gamma E_0\, e^{-j\beta_1(x-z)/\sqrt{2}} = \left[-\mathbf{a}_y - j\frac{\mathbf{a}_x + \mathbf{a}_z}{\sqrt{2}}\right]E_0\, e^{-j\beta_1(x-z)/\sqrt{2}},$$

   and this is *left-hand circular polarization.*