Name:
Student no:

Answer all four questions (in English, Finnish, or Swedish). Using a calculator is allowed, but all memory must be cleared! **Please return also this problem paper.** Remember to write your name and student number above.

1. Describe the cascade control of a DC motor drive. Draw also the block diagram of the control system, label the signals in the diagram, and describe the tasks of the blocks.

   Solution:

   See lectures, exercises, and readings.

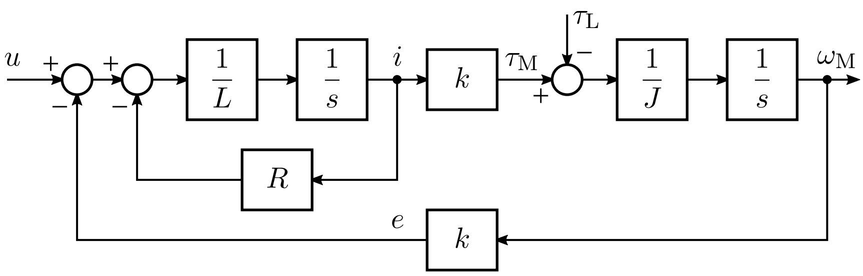
2. Answer *briefly* to the following questions:
   (a) What is the basic function of an electric drive?
   (b) Why a speed reduction gear is often used in electric drives?
   (c) What is the concept of time-scale separation (in the context of motor drives)?

   Solution:

   See lectures, exercises, and readings.

3. The block diagram of a DC machine is shown in the figure below. Derive the transfer function from the voltage to the rotor speed

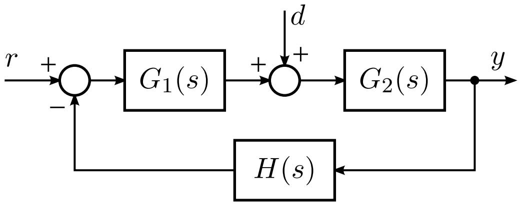
$$G(s) = \frac{\omega_\mathrm{M}(s)}{u(s)}$$



   Solution:

   This problem is a subset of Problem 1 in Exercise 3.

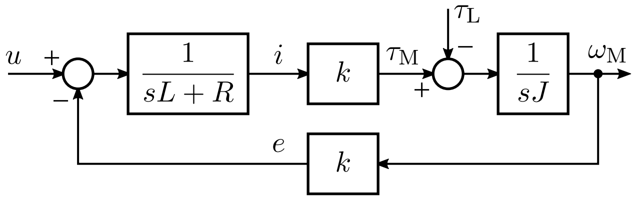
   Consider a closed-loop system shown in the figure.



The following equations hold for the closed-loop transfer functions:

$$\frac{y(s)}{r(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{1}$$

It is relatively easy to derive these equations if one has forgotten them. Using (1), the block diagram given in the problem is first transformed to the following form:



Using (1), we can write the transfer function from the voltage to the speed as

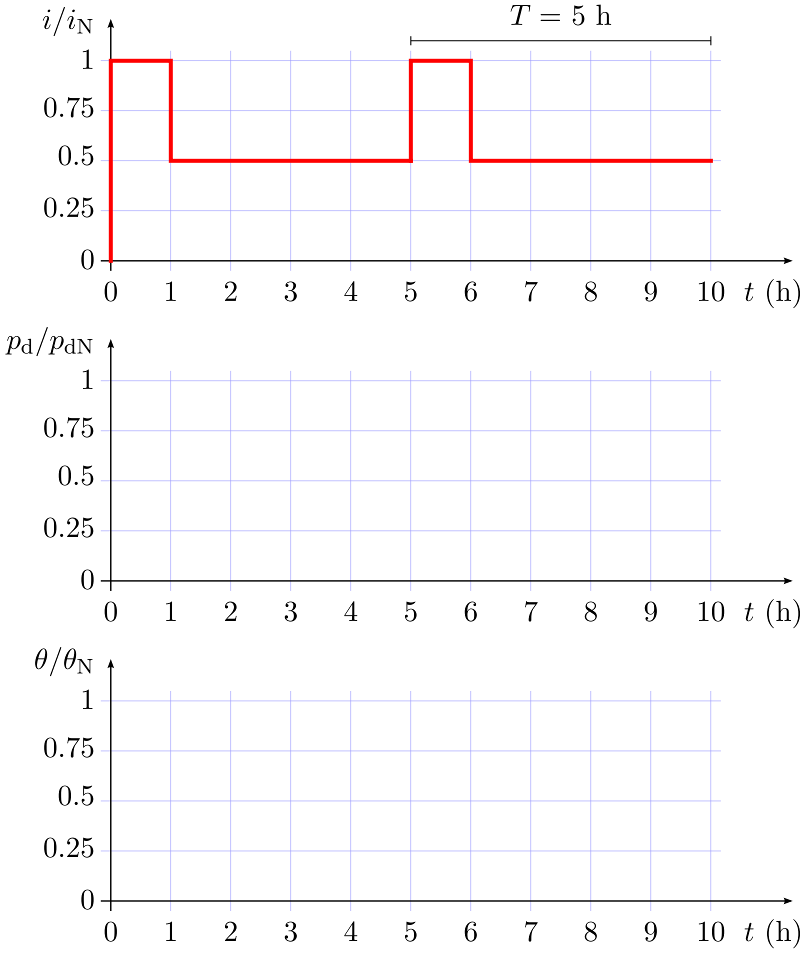
$$G(s) = \frac{\omega_M(s)}{u(s)} = \frac{\frac{1}{sL+R}k\frac{1}{sJ}}{1 + \frac{1}{sL+R}k\frac{1}{sJ}k} = \frac{\frac{k}{JL}}{s^2 + s\frac{R}{L} + \frac{k^2}{JL}}$$

4. (a) A DC motor is used in a periodic duty, whose cycle length is $T = 5$ h. As shown in the figure below, the armature current is $i = i_\mathrm{N}$ for 1 hour and $i = 0.5i_\mathrm{N}$ for 4 hours during each cycle. Calculate the rms current i_rms over the cycle.

   (b) In the graph below, draw the waveform for the instantaneous resistive losses p_d as a function of time.

   (c) The thermal model of the motor can be assumed to be a first-order system, having the time constant $T_\mathrm{th} = 15$ min. The motor is cold in the beginning, i.e., the initial temperature rise $\theta = 0$ at $t = 0$. What is the value of the instantaneous temperature rise at $t = 1$ h?
   Tip: The step response is $\theta(t) = \theta_0 + (\theta_\infty - \theta_0)\left(1 - \mathrm{e}^{-t/T_\mathrm{th}}\right)$, where $\theta_0 = \theta(0)$.

   (d) In the graph below, sketch the waveform for the instantaneous temperature rise θ. Is the motor suitable for this periodic duty?



Solution:

(a) Using the given current waveform, the rms current is

$$i_\mathrm{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 \mathrm{d}t} = \sqrt{\frac{i_\mathrm{N}^2 \cdot 1\ \mathrm{h} + (0.5i_\mathrm{N})^2 \cdot 4\ \mathrm{h}}{5\ \mathrm{h}}} = 0.63i_\mathrm{N}$$

(b) The instantaneous resistive losses

$$p_\mathrm{d} = Ri^2$$

are proportional to the square of the current. The losses can be normalized by the rated losses $p_\mathrm{dN} = Ri_\mathrm{N}^2$, giving

$$\frac{p_\mathrm{d}}{p_\mathrm{dN}} = \left(\frac{i}{i_\mathrm{N}}\right)^2$$

The corresponding waveform is shown below.

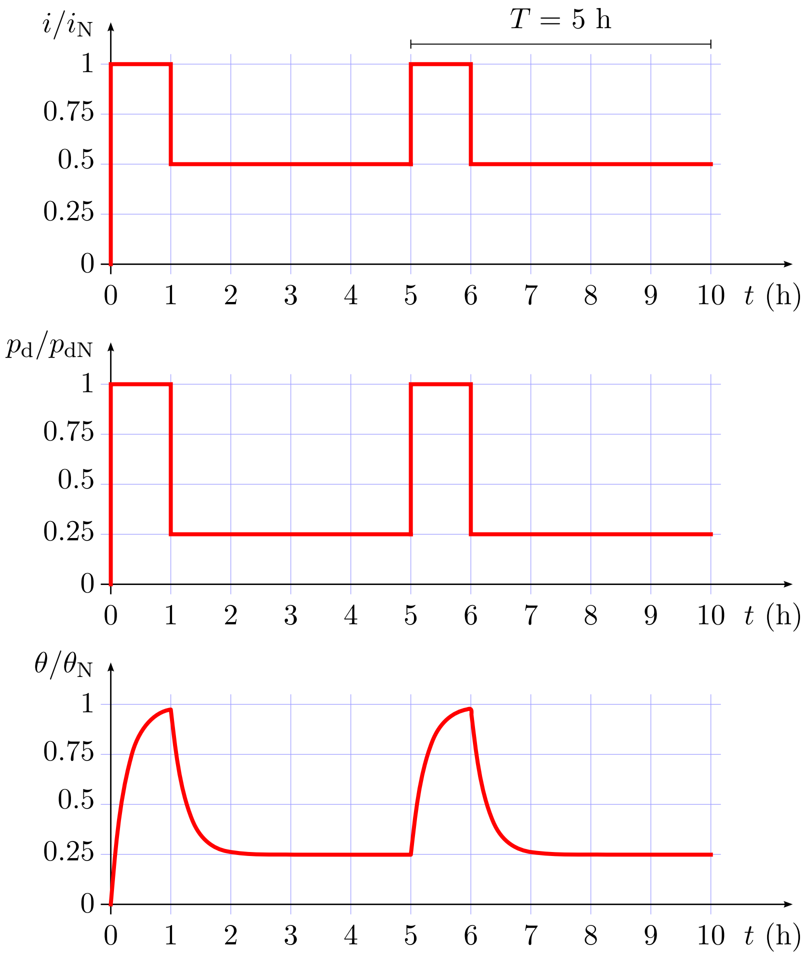
(c) The thermal model can be assumed to be a first-order system. Therefore, the step response of the temperature rise is

$$\theta(t) = \theta_0 + (\theta_\infty - \theta_0)\left(1 - \mathrm{e}^{-t/T_\mathrm{th}}\right) = \theta_\mathrm{N}\left(1 - \mathrm{e}^{-t/T_\mathrm{th}}\right)$$

In this case, $\theta_0 = 0$ and $\theta_\infty = \theta_\mathrm{N}$. Therefore, the temperature rise at $t = 1$ h is $\theta = (1 - \mathrm{e}^{-60/15})\theta_\mathrm{N} = 0.98\theta_\mathrm{N}$.

(d) The waveform for the instantaneous temperature rise is drawn in the graph below. The motor is suitable for this periodic duty since the maximum temperature rise is close to the rated temperature rise.

If the cycle length is not much shorter than the thermal time constant of the motor, the motor should be selected based on the maximum temperature rise (instead of the average temperature rise). In this example, the average temperature rise is only $0.63\theta_\mathrm{N}$.


T = 5 h
i/i_N
1
0.75
0.5
0.25
0
0 1 2 3 4 5 6 7 8 9 10 t (h)
p_d/p_dN
1
0.75
0.5
0.25
0
0 1 2 3 4 5 6 7 8 9 10 t (h)
θ/θ_N
1
0.75
0.5
0.25
0
0 1 2 3 4 5 6 7 8 9 10 t (h)