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Department of Computer Science
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CS-C2160 Theory of Computation

Spring 2023

Exam Wed 18 Oct 2023, 1–4 p.m.

You are allowed to bring with you a single one-sided A4 “cheat sheet”, **personally handwritten by you**. (NO photocopies, NO printouts, NO computer type-set text.) Please include your name and student ID at the top of the cheat sheet, and return it together with your answer sheets at the end of the exam.

Note: If you have not completed your computerised home assignments, your exam will not be graded.

1. (a) Design a finite automaton that recognises the language

$$L = \{w \in \{0, 1\}^* \mid \text{the number of 1's in } w \text{ is a multiple of 2 or 3 (or both)}\}.$$

(Here zero is taken to be a multiple of any number.) 8 points

- (b) Give a regular expression that describes the language L in part (a). 7 points

2. (a) Design a context-free grammar for the language

$$L = \{[1^j 0^k]^i \mid i, j, k \geq 0, j \geq k\}.$$

Draw the corresponding parse trees for the words $[[1]]$ and 110 . 8 points

- (b) Prove (precisely!) that the language L discussed in part (a) is not regular. 7 points

3. Give brief but precise justifications, based on material presented on the course, for the following statements:

- (a) All regular languages are context-free. 4 points
(b) All context-free languages are decidable. 4 points
(c) All decidable languages are semidecidable. 4 points
(d) There exist languages that are not semidecidable. 4 points

(No proofs or other constructions are needed, just explanations based on the relevant concepts and results on the course.)

4. Design a deterministic single-tape Turing machine that replaces an input string of the form $a^i b^j$, $i \geq j \geq 0$, given on its tape, by the string $A^i B^j C^k$, where $k = i - j$. Your machine does not need to check the validity of its input, i.e. you may assume that any given input string is of the indicated form. (Present the Turing machine preferably as a state diagram rather than a transition table.) Show the computation sequences (“runs”) of your machine on inputs aab , ab , and a . 14 points

Total 60 points