Matrix Algebra
MS-A0001
Brzuska/Karanko/Hakula
Course Exam, 5.12.2023

**This exam is for those participating on continuous assessment.** Every question carries an equal weight, similarly every part of a question carries an equal weight, unless otherwise specified. There are four problems on this exam. Calculators are not permitted.

PROBLEM 1 Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad b = \begin{pmatrix} 6 \\ 3 \\ 1 \end{pmatrix}.$$

(a) Show that the columns of $A$ are linearly independent. (b) Show that $x = (1\ 2\ 3)^T$ is a solution of $Ax = b$.

PROBLEM 2 Solve the linear system of equations $Ax = b$ using some suitable $LU$-factorization, where

$$A = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 4 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix}.$$

PROBLEM 3 Let

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ \alpha \end{pmatrix},$$

where $\alpha \in \mathbb{R}$. Find all possible solutions (if any) of $Ax = b$.

PROBLEM 4 (4p) Find a real and symmetric matrix $A$, whose eigenvalues are $\lambda_1 = 1$, $\lambda_2 = 1/2$ and the corresponding eigenvectors are

$$x_1 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}.$$

(2p) Find the limit

$$\lim_{k \to \infty} A^k.$$