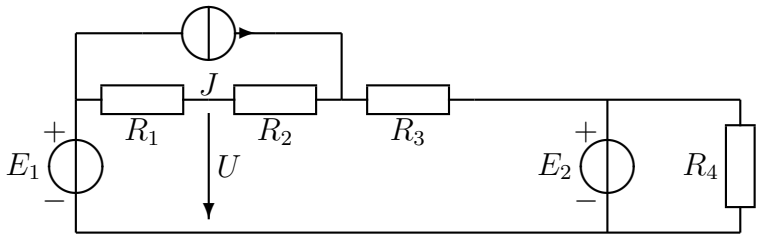
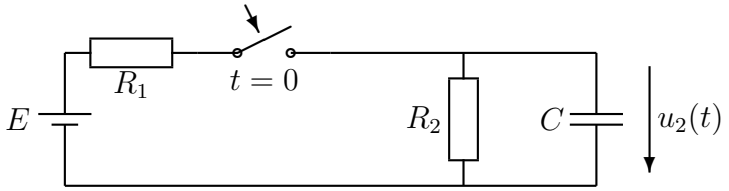
ELEC–C4210 SÄHKÖTEKNIIKKA JA ELEKTRONIIKKA

1. välikoe 18.10 2023. Saat vastata vain neljään tehtävään!
1. mellanförhör 18.10 2023. Du får endast besvara fyra frågor!

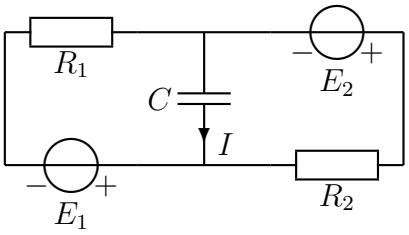
1. Laske jännite U. Beräkna spänningen U.
$R_1 = 3\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 4\ \Omega$, $R_4 = 2\ \Omega$, $J = 3$ A, $E_1 = 6$ V, $E_2 = 12$ V.



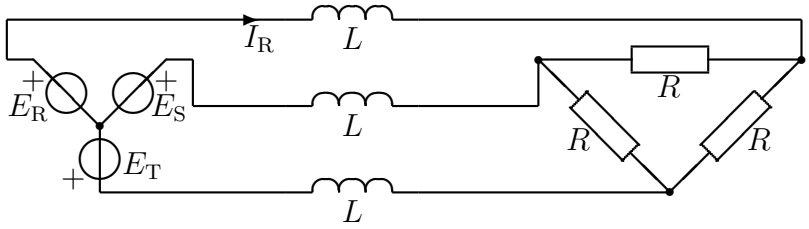
2. Kytkin suljetaan hetkellä $t = 0$. Kuinka kauan kestää, että $u_2(t)$ nousee 90 prosenttiin maksimiarvostaan?
Spänningskällan kopplas vid $t = 0$. Hur lång tid tar det för $u_2(t)$ att nå 90 % av sitt maximala värde? $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$, $C = 0{,}22$ F, $U_{C0} = 0$ V.



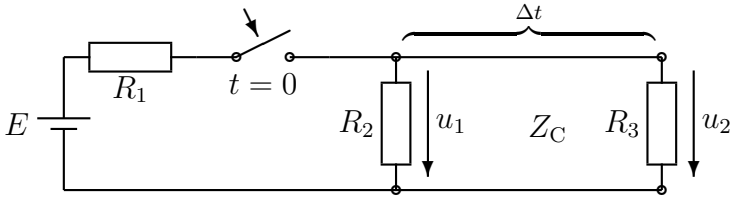
3. Laske virta I. Beräkna strömmen I.
$R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$, $C = 0{,}05$ F, $\omega = 10\ \frac{\text{rad}}{\text{s}}$, $E_1 = 3\angle 0°$ V, $E_2 = 4\text{j}$ V.



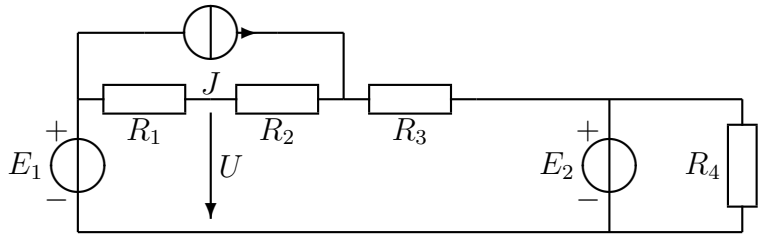
4. Laske virta I_R. Beräkna strömmen I_R.
$E_\text{R} = 230\angle 0°$ V, $R = 120\ \Omega$, $L = 0{,}1$ H, $\omega \approx 300\ \frac{\text{rad}}{\text{s}}$.



5. Jos lasket tämän, jätä yksi tehtävistä 1–4 pois. Om du svarar påden 5., lämna en av 1–4 bort! Laske jännite $u_2(t_1)$. Beräkna spänningen $u_2(t_1)$. $\rho_1 = \frac{R_{12} - Z_\text{C}}{R_{12} + Z_\text{C}}$, $R_{12} = \frac{R_1 R_2}{R_1 + R_2}$
$R_1 = 100\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 100\ \Omega$, $Z_\text{C} = 50\ \Omega$, $E = 10$ V, $t_1 = 4\Delta t$.



1. Laske jännite U. $R_1 = 3\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 4\ \Omega$, $R_4 = 2\ \Omega$, $J = 3$ A, $E_1 = 6$ V, $E_2 = 12$ V.



$$-E_1 + R_1 I + U = 0 \Rightarrow I = \frac{E_1 - U}{R_1} \tag{1}$$

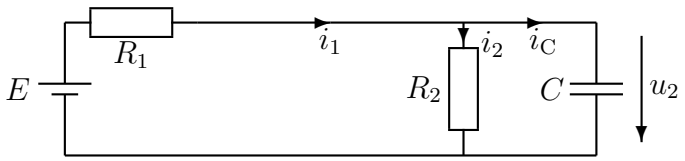
$$-U + R_2 I + R_3(I + J) + E_2 = 0 \Rightarrow -U + (R_2 + R_3)I + R_3 J + E_2 = 0 \tag{2}$$

$$-U + (R_2 + R_3)\frac{E_1 - U}{R_1} + R_3 J + E_2 = 0 \tag{3}$$

$$-U + (R_2 + R_3)\frac{-U}{R_1} + (R_2 + R_3)\frac{E_1}{R_1} + R_3 J + E_2 = 0 \tag{4}$$

$$U = \frac{\frac{R_2+R_3}{R_1} E_1 + R_3 J + E_2}{1 + \frac{R_2+R_3}{R_1}} = 12\ \mathrm{V} \tag{5}$$

2. Kytkin suljetaan hetkellä $t = 0$. Kuinka kauan kestää, että $u_2(t)$ nousee 90 prosenttiin maksimiarvostaan? $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$, $C = 0{,}22$ F, $U_{\mathrm{C}0} = 0$ V.



$$-E + R_1(\underbrace{i_2}_{\frac{u_2}{R_2}} + \underbrace{i_\mathrm{C}}_{C\frac{\mathrm{d}u_2}{\mathrm{d}t}}) + u_2 = 0 \tag{6}$$

$$\left(1 + \frac{R_1}{R_2}\right) \underbrace{u_2}_{B + A\mathrm{e}^{-t/\tau}} + R_1 C \underbrace{\frac{\mathrm{d}u_2}{\mathrm{d}t}}_{0 - \frac{A}{\tau}\mathrm{e}^{-t/\tau}} = E \tag{7}$$

$$\underbrace{\left(1 + \frac{R_1}{R_2}\right) B}_{munat} + \underbrace{\left(1 + \frac{R_1}{R_2}\right) A}_{jauhot}\mathrm{e}^{-t/\tau} = \underbrace{E}_{munat} + \underbrace{R_1 C \frac{A}{\tau}}_{jauhot}\mathrm{e}^{-t/\tau} \tag{8}$$

$$mnt:\ B = \frac{R_2}{R_1 + R_2} E = \frac{2}{3} E \qquad jht:\ \tau = \frac{R_1 R_2}{R_1 + R_2} C = 0{,}44\ \mathrm{s} \tag{9}$$

$$U_{\mathrm{C}0} = u_2(0) = B + A\mathrm{e}^{-0/\tau} = B + A \Rightarrow A = -B = -\frac{2}{3} E \tag{10}$$

$$u_2 = B + A\,\mathrm{e}^{-t/\tau} = \frac{R_2}{R_1 + R_2} E \left(1 - \mathrm{e}^{-t/\tau}\right) \tag{11}$$

$$u_{2\mathrm{MAX}} = \frac{R_2}{R_1 + R_2} E\ (1 - 0) \tag{12}$$

$$1 - \mathrm{e}^{-t/\tau} = 0{,}9 \Rightarrow \mathrm{e}^{-t/\tau} = 0{,}1 \tag{13}$$

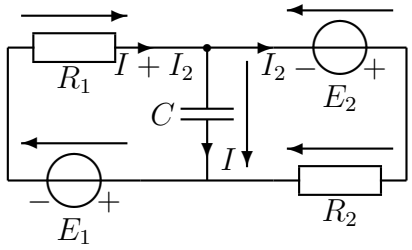
Tämän voi ratkaista kokeilemalla:

$$\mathrm{e}^{-0{,}9/0{,}44} = 0{,}129 \tag{14}$$

$$\mathrm{e}^{-1{,}0/0{,}44} = 0{,}103 \Rightarrow t \approx 1{,}0\ \mathrm{s} \tag{15}$$

$$\mathrm{e}^{-1{,}1/0{,}44} = 0{,}082 \tag{16}$$

3. Laske virta I. $R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$, $C = 0{,}05$ F, $\omega = 10\ \frac{\text{rad}}{\text{s}}$, $E_1 = 3\angle 0°$ V, $E_2 = 4\text{j}$ V.



$$+E_1 + R_1(I + I_2) + \frac{1}{\text{j}\omega C}\, I = 0 \tag{17}$$

$$-\frac{1}{\text{j}\omega C}\, I - E_2 + R_2 I_2 = 0 \Rightarrow I_2 = \frac{E_2 + \frac{1}{\text{j}\omega C}\, I}{R_2} \tag{18}$$

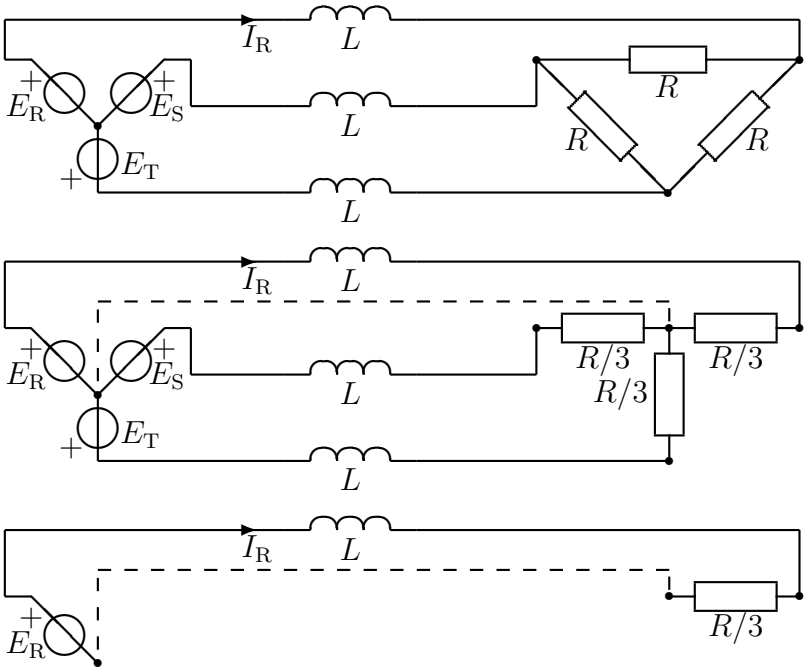
$$E_1 + \left(R_1 + \frac{1}{\text{j}\omega C}\right) I + R_1\, \frac{E_2 + \frac{1}{\text{j}\omega C}\, I}{R_2} = 0 \tag{19}$$

$$I = \frac{-E_1 - \frac{R_1}{R_2}\, E_2}{R_1 + \frac{1}{\text{j}\omega C} + \frac{R_1}{R_2}\, \frac{1}{\text{j}\omega C}} = -\text{j}\omega C\, \frac{E_1 + \frac{R_1}{R_2}\, E_2}{\text{j}\omega C R_1 + 1 + \frac{R_1}{R_2}} \tag{20}$$

$$= -\text{j}0{,}5\, \frac{3 + 4\text{j}}{\text{j} + 1 + 1} = \frac{4 - 3\text{j}}{4 + 2\text{j}} = \frac{\sqrt{25}\angle -36{,}9°}{\sqrt{20}\angle 26{,}6°} = 1{,}12\angle -63{,}4°\ \text{A} \tag{21}$$

$$(I = 0{,}5 - \text{j}) \tag{22}$$

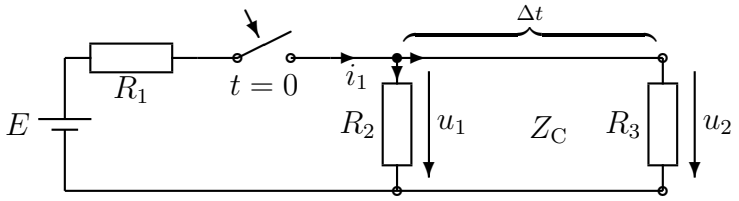
4. Laske I_R. $E_\text{R} = 230\angle 0°$ V, $R = 120\ \Omega$, $L = 0{,}1$ H, $\omega \approx 300\ \frac{\text{rad}}{\text{s}}$.



$$-E_\text{R} + \text{j}\omega L I_\text{R} + \frac{R}{3}\, I_\text{R} = 0 \tag{23}$$

$$I_\text{R} = \frac{E_\text{R}}{\frac{R}{3} + \text{j}\omega L} = \frac{230}{40 + \text{j}30} = \frac{23}{4 + \text{j}3} = 4{,}6\angle -36{,}87°\ \text{A} \tag{24}$$

5. Laske jännite $u_2(t_1)$. $\rho_1 = \frac{R_{12}-Z_\mathrm{C}}{R_{12}+Z_\mathrm{C}}$, $R_{12} = \frac{R_1 R_2}{R_1+R_2}$
$R_1 = 100\ \Omega$, $R_2 = 100\ \Omega$, $R_3 = 100\ \Omega$, $Z_\mathrm{C} = 50\ \Omega$, $E = 10$ V, $t_1 = 4\Delta t$.



$$-E + R_1 i_1 + u_1(0) = 0 \Rightarrow -E + R_1\left(\frac{u_1(0)}{R_2} + \frac{u_1(0)}{Z_\mathrm{C}}\right) + u_1(0) = 0 \tag{25}$$

$$\Rightarrow u_1(0) = \frac{E}{\frac{R_1}{R_2} + \frac{R_1}{Z_\mathrm{C}} + 1} = \frac{E}{4} \tag{26}$$

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 50\ \Omega \tag{27}$$

$$\rho_1 = \frac{R_{12} - Z_\mathrm{C}}{R_{12} + Z_\mathrm{C}} = 0 \tag{28}$$

$$\rho_2 = \frac{R_3 - Z_\mathrm{C}}{R_3 + Z_\mathrm{C}} = \frac{1}{3} \tag{29}$$

$$\tau_2 = 1 + \rho_2 = \frac{4}{3} \tag{30}$$

$$u_2(t \geq \Delta t) = \tau_2 u_1(0) = \frac{E}{3} = 3{,}33\ \mathrm{V} \tag{31}$$

2,5 V korkea aalto heijastuu johdon loppupäästä hetkellä Δt ja saavuttaa johdon alkupään hetkellä $2\Delta t$. Siellä ei enää tapahdu uutta heijastusta, joten lopputila on saavutettu; lopputilanne olisi voitu laskea suoraan vastuksista jättämällä siirtojohto pois välistä.