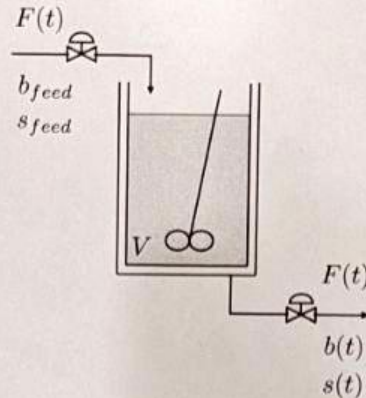


**Exercise Q1.** Biochemical reactors are process units used in the production of a wide variety of intermediate and final products, including food, beverages, and pharmaceuticals. Similarly to chemical reactors their models are based on material balances. The simplest bioreactors consists of two components: biomass and substrate. The biomass consists of cells that consume substrate, as in fermentation where cells consume sugar and produce alcohol.



Consider a perfectly mixed and continuous biochemical reactor with constant liquid volume  $V$  (litres), see figure. Let  $b(t)$  and  $s(t)$  be the biomass and substrate concentrations in time  $(t)$ , respectively: That is,  $b(t) = \frac{\text{mass of cells at } t}{\text{volume}}$  and  $s(t) = \frac{\text{mass of substrate at } t}{\text{volume}}$ . Let  $b_{feed} = 0$  and  $s_{feed} = 4$  be the feed concentrations (in grams per litre), we shall assume them to be constant in time. Let  $F(t)$  be the volumetric feed flow-rate (in litres per hour).

(Q1.1) Write the material balances for biomass and substrate assuming the following:

↪ Instantaneous (at  $t$ ) rate of biomass generation

$$\underbrace{r_b(t)}_{+} = \frac{\text{mass of cells generated}}{\text{volume} \times \text{time}} \quad \frac{m}{V} \quad (1a)$$

$$= \mu(t)b(t) \quad (1b)$$

↪ Instantaneous (at  $t$ ) rate of substrate consumption

$$\underbrace{r_s(t)}_{-} = \frac{\text{mass of substrate consumed}}{\text{volume} \times \text{time}} \quad (2a)$$

$$= 0.4 \times (\mu(t)b(t)) \quad (2b)$$

where the growth rate coefficient  $\mu(t)$  is assumed to be of the Monod type: That is,

$$\mu(t) = \mu_{\max} \frac{s(t)}{k_m + s(t)}, \quad \text{with} \quad \begin{cases} \mu_{\max} = 0.53 \text{ [ hr}^{-1}\text{]} \\ k_m = 0.12 \text{ [ g l}^{-1}\text{]} \end{cases}$$

(Q1.2) Assuming that we measure the substrate concentration, list the state variables ( $x$ ) and their dimensionality ( $N_x$ ), the control variables ( $u$ ) and their dimensionality ( $N_u$ ), the output variables  $y$  and their dimensionality ( $N_y$ ), and the model parameters ( $\theta_x$  and  $\theta_y$ ). Then, write the state-space model (dynamics and measurement equations) for this problem.

(Q1.3) Assume i) a steady-state dilution rate  $D_{ss} = F_{ss}/V = 0.15 \text{ [hr}^{-1}\text{]}$  associated to a steady-state feed flow-rate  $u_{ss} = F_{ss}$  and a volume  $V$  of your choice; and ii) a steady state  $x_{ss} = (b_{ss} = 1.5374, s_{ss} = 0.1565)$ , then determine the linear time-invariant (LTI) approximation of the nonlinear state-space model: That is, determine the matrices ( $A, B, C, D$ ) from the Jacobians of dynamics and measurement equation evaluated at the equilibrium ( $x_{ss}, u_{ss}$ ).

**Exercise Q2.** The linearisation of dynamics and measurement equation at a steady-state point ( $x_{SS}, u_{SS}$ ) of a process led to the linear time-invariant (LTI) state-space model below:

$$\dot{x}(t) = \begin{bmatrix} 1 & -3 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) \quad (3a)$$

$$y(t) = \begin{bmatrix} 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \end{bmatrix} u(t) \quad (3b)$$

(Q2.1) Determine the stability of the steady-state point / system. ✓

(Q2.2) Determine the controllability matrix of the system (Q2.2a), then use it to establish whether the system is controllable or not (Q2.2b). ✓

(Q2.3) Determine the observability matrix of the system (Q2.3a), then use it to establish whether the system is controllable or not (Q2.3b). ✓

↖ observable?

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Give-me-points table:

- Q1: Max 65/100 points
  - Q1.1: 20/100 points
  - Q1.2: 15/100 points
  - Q1.3: 30/100 points
- Q2: Max 35/100 points
  - Q2.1: 05/100 points
  - Q2.2: 15/100 points
  - Q2.3: 15/100 points

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This is an open-book examination. In addition to pencil/pen, eraser and other writing material, the use of own printed copies of the course material and personal notes is allowed.