This examination consists of **four questions** (across **three** pages) of which each is graded on a scale from **zero to 25 points.**

Please **justify** your answers and write your **name, student registration number**, and **degree programme** clearly on each answer sheet.

Only a **basic calculator** is allowed as extra equipment.
For numerical answers, two decimal are enough.

1 Question I (25 points)

A new potential investor has been reading much news regarding financial marketing and investments. Among the new definitions they have found, the following caught their attention the most:

The ***interest rates*** *and* ***inflation*** *have an intrinsic behaviour. Both are directly related; you cannot express one without considering the other. In addition, all different forms of interest rates (such as* ***spot, forward, short****) and financial instruments (****bonds*** *and* ***annuities****) should always be considered in light of how inflation behaves in such period.*

Answer the following questions:

1. **Provide** a simple definition for **inflation, interest rates** and **bonds.** (9 points).
   Please provide your answer within 3 lines of text per definition.

2. **Boostrapping** and **repetition** are two methods to calculate spot rates. Is this statement **correct**? **Justify** your answer. (3 points)
   Please provide your answer within 3 lines of text.

3. What is the difference between **perpetual** and **finite length** annuities? (3 points) Please provide your answer within 3 lines of text.

4. Considering an **interest rate** equal to $r = 3\%$ and **inflation** of $f = 1.5\%$. What is the **real interest rate**? (3 points)

5. Considered a **2-year bond** with price $P = 2000$ and an **coupon payment** $C = 500$ and **face value** of $F = 800$. If the spot rate for one year

is **equal** to the real interest (solution from the previous task), what is the **spot rate** for **2** years? (7 points)

2 Question II (25 points)

Suppose the only stocks in the market are those in Table 1. The correlation between the stocks' returns is $\rho_{AB} = 1/2$. There are also risk-free government obligations in the market. The assumptions of the CAPM (Capital Asset Pricing Model) hold and the expected return of the market portfolio is 7.5%.

Table 1: Assets on the market.

Stock	Number of stock	Price	Expected return	Std. of returns
A	100	20€	?	0.15
B	200	15€	0.04	0.10

1. Which is value for **correlaction** between two assets that guarantee that they do not interfere with each other? (2 points)
   Please provide your answer within 2 lines of text.
2. What is the **expected return** of stock A? (8 points)
3. What is the **standard deviation** of the market portfolio? (8 points)
4. What are the β's of the stocks? (7 points)

3 Question III (25 points)

Harry Markowitz is a Nobel Prize Winner economist responsible for a model where the portfolio of an efficient frontier can be found.

Answer the following questions:

1. **Diversification** is the principle where correlated assets are undesirable for an efficient portfolio. Is this statement **correct**? **Justify** this statement. (4 points)
   Please provide your answer within 4 lines of text.
2. With random returns, the variance offers a measure of risk, and the expected value highlights the measure of reliability. Is this statement **correct**? (5 points)
   Please provide your answer within 5 lines of text.
3. CAPM is an extension of the one-fund theorem where the fund F becomes the market M. How does this affect the investor? Is this model the same as **Markowitz model**? (8 points)
   Please provide your answer within 8 lines of text.

4. The most important **parameters** from CAPM (below) is β. What does β_i represent? Is β a **useful** guide for investment selection? **Justify** your answer. (8 points)
   Please provide your answer within 8 lines of text.

4 Question IV (25 points)

A stock is currently valued at $S(0) = 50$ € , and its relative monthly price changes up and down are described by the parameters $u = 1.20$ and $d = 0.83$, respectively. The price of this stock moves upward each month with probability $p = 0.65$. The annual risk-free rate is $r_f = 5\%$.

1. Based on a **binomial lattice**, what is the price of an **American put option** on this stock, assuming that this option expires in **three** months with a strike price K of 65 €? (12 points)

2. With what **probabilities** are each of the **end states** of this **binomial lattice** reached? (10 points)

3. The **binomial lattice** used in previous questions is an **additive** model. Is this sentence **correct**? Justify your answer. (3 points)

   Please provide your answer within 2 lines of text.

A Useful Equations

Real interest rate:

$$1 + r_0 = \frac{1+r}{1+f}$$

Spot rates:

$$P = \frac{C}{1+s_1} + \frac{C+F}{(1+s_2)^2}$$

Forward rates (yearly compounding):

$$f_{ij} = \left(\frac{(1+s_j)^j}{(1+s_i)^i}\right)^{\frac{1}{j-i}} - 1$$

Expected values:

$$r_m = w_a r_a + w_b r_b$$

Covariance:

$$\sigma_m^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = w^T \sum w$$

where:
$w = [w_1, \ldots, w_n]$ and $\sum =$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_n^2 \end{bmatrix}$$

$$cov(r_A, r_m) = w_A \sigma_A^2 + w_B \rho_{AB} \sigma_A \sigma_B$$

Beta:

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

Options:

$$R = 1 + \frac{r_f}{12}$$

$P = \max\{\frac{1}{R}(qP_u + (1-q)P_d, K - S\}$

$P = \max\{K - S, 0\}$