Department of Mathematics and Systems Analysis
Aalto University, School of Science

Metsalo

MS-A0210 Mathematics 1

Final exam, 07.09.2017

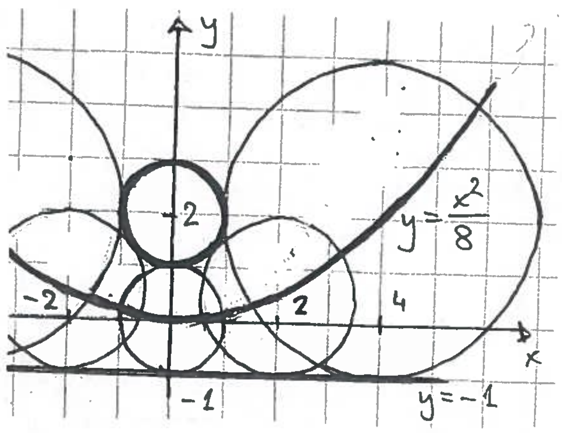
Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Degree Programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

Calculators are not allowed. The examination time is 3 hours.
Ask, if You suspect typos in the text!

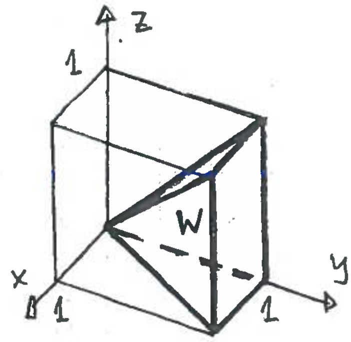
1. $g(x, y, z) = \exp(2\sqrt{x+y} + y - z)$, so $g(7, 9, 17) = \exp(2\sqrt{16} + 9 - 17) = e^0 = 1$.
   Approximate the value $g(7.03, 8.99, 17.02)$ using linear approximation.

2. The function $h(x, y) = \frac{x-2y}{1+x^2+2y^2}$ has limit 0, as $\sqrt{x^2+y^2} \to \infty$. Determine the maximum and minimum value of $h(x, y)$ in the upper half plane $y \geq 0$.

3. We study the family of circles with center on the parabola $y = x^2/8$ and tangent to the line $y = -1$ (see the upper figure to the right).
   The equation for this family of circles is
   $(x-c)^2 + (y - c^2/8)^2 = (1 + c^2/8)^2 \Leftrightarrow$
   $f(x, y, c) = x^2 - 2cx + c^2 + y^2 - c^2y/4 - 1 - c^2/4 = 0$.
   The line $y = -1$ is ofcourse an envelope for this family of circles. Show that the circle $x^2 + (y-2)^2 = 1$ (except for the point $(0, 3)$) is also an envelope to this family of circles.



4. Calculate $\iiint_W \sin(\pi y^3)dV$, where W is the pyramid in the lower figure to the right with vertices $(0,0,0), (0,1,0), (1,1,0), (1,1,1)$ and $(0,1,1)$.
   (W is hence a third of the unit cube $0 \leq x, y, z \leq 1$, but that has nothing to do with the problem.)



5. Calculate the area of the part of the saddle surface $z = xy$, which is inside the right circular cylinder $x^2 + y^2 = 2$.
   (Hint: The area of the surface can not be less than the area of its projection onto a (coordinate) plane.)

Useful (?) formulas:
$(a+b)^2 = a^2 + 2ab + b^2$, $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
$\cos^2 t + \sin^2 t = 1$, $\cos^2 t = (1 + \cos(2t))/2$, $\sin^2 t = (1 - \cos(2t))/2$,
$\sin(2t) = 2 \sin t \cos t$, $\cos(2t) = \cos^2 t - \sin^2 t = 2\cos^2 t - 1 = 1 - 2\sin^2 t$.