

Helsinki University of Technology
Mat-1.461 Mathematics I

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Second mid-term Exam. 8/November/2004

Please write on sheet on separate lines:

1) course title, date

2) student number + letter, IN BLOCK LETTERS surname underlined, all given names

3) signature

1. (a) Evaluate the following limits

i. $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x}$

ii. $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 3x} - x)$

(b) Find a nonzero value for the constant k that makes

$$\begin{cases} \frac{\tan(kx)}{x}, & x < 0 \\ 3x + 3k^2, & x \geq 0 \end{cases}$$

continuous at $x = 0$

2. (a) Find the equation of the tangent line to the curve

$$\sin(xy) = y$$

at the point $(\pi/3, 1)$.

(b) Show that the equation

$$x^3 - 4x + 1 = 0$$

has at least one real solution in the interval $[1, 2]$.

(c) Use an appropriate local linear approximation to estimate the value of $\cot(46^\circ)$.

3. (a) Locate the relative extrema of the function

$$f(x) = \frac{1}{2}x + \cos x$$

in the interval $(0, 2\pi)$.

(b) Locate all points on the curve $x^2 - y^2 = 1$ closed to $(0, 2)$.

4. Find the Taylor polynomial of order $n = 4$ for $f(x) = \sin \pi x$ about $x = \frac{1}{2}$.