Aalto-yliopiston perustieteiden korkeakoulu
Matematiikan ja systeemianalyysin laitos
MS-A0101 Differentiaali- ja integraalilaskenta 1 (TFM)   Alestalo
MS-A0104 Differentiaali- ja integraalilaskenta 1 (ELEC2)
MS-A0106 Differentiaali- ja integraalilaskenta 1 (ENG2)
MS-A0107 Differentiaali- ja integraalilaskenta 1 (CHEM)

Tentti 25.5.2018

No calculators nor table books are allowed.
Choose five problems. You can also try to solve all six problems, but only the best five are counted.

1. We consider solving numerically the equation $f(x) = 0$ by using Newton's method.
   a) Explain how the recursion formula

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

   is obtained by using the tangent line of the graph.
   b) Calculate the first iteration step x_1 for the equation $x^4 - 3 = 0$, when $x_0 = 1$.

2. a) For which values $x \in \mathbf{R}$ does the power series

   $$\sum_{k=1}^{\infty} k(x-3)^k$$

   converge?
   b) We assume that the terms a_k of the series

   $$\sum_{k=1}^{\infty} a_k$$

   satisfy $\sqrt[k]{|a_k|} \le 1/5$ for all k. Show that the series is convergent.

3. a) Find the Maclaurin polynomial $P_3(x)$ of the function $f(x) = \sin(3x)$ (Maclaurin polynomial = Taylor polynomial with $x_0 = 0$).
   b) Calculate the limit

   $$\lim_{x \to 0} \frac{\sin^2(3x)}{1 - \cos x}.$$

Please turn!

4. Evaluate the integrals

$$\int_0^{\sqrt{\pi/2}} x \cos(x^2)\, dx \quad \text{and} \quad \int_0^4 e^{\sqrt{x}}\, dx.$$

Hint: For the latter, start by substituting $x = t^2$.

5. Solve the differential equation

$$y' = 6\sqrt{y}$$

with the initial condition $y(0) = 9$.

6. Find the solution of

$$y'' + 2y' + 2y = 0$$

satisfying the initial conditions $y(0) = 2$, $y'(0) = -3$.

Additional information:

Some values of trigonometric functions:

α	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin(\alpha)$	$-1/\sqrt{2}$	$-1/2$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0
$\cos(\alpha)$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1
$\tan(\alpha)$	-1	$-1/\sqrt{3}$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$-$	0

Some Maclaurin approximations:

$$e^x \approx 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n$$

$$\ln(1+x) \approx x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots + \frac{(-1)^{n+1}}{n}x^n$$

$$\sin x \approx x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots + \frac{(-1)^n}{(2n+1)!}x^{2n+1}$$

$$\cos x \approx 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots + \frac{(-1)^n}{(2n)!}x^{2n}$$